Now,

$$\alpha u+\beta v = \alpha(\alpha_1 u_1+\alpha_2 u_2+\dots+\alpha_n u_n+\beta_1 v_1+\beta_2 v_2+\dots+\beta_n v_n)$$
$$+\beta(c_1 u_1+c_2 u_2+\dots+c_n u_n+d_1 v_1+d_2 v_2+\dots+d_n v_n)$$

$$= \alpha\alpha_1 u_1+\alpha\alpha_2 u_2+\dots+\alpha\alpha_n u_n+\alpha\beta_1 v_1+\alpha\beta_2 v_2+\dots+\alpha\beta_n v_n$$
$$+\beta c_1 u_1+\beta c_2 u_2+\dots+\beta c_n u_n+\beta d_1 v_1+\beta d_2 v_2+\dots+\beta d_n v_n$$

$$\cancel{= p_1 u_1+p_2 u_2+\dots+p_n u_n+q_1 v_1+q_2 v_2+\dots+q_n v_n}$$

$$= (\alpha\alpha_1+\beta c_1)u_1+(\alpha\alpha_2+\beta c_2)u_2+\dots+(\alpha\alpha_n+\beta c_n)u_n$$
$$+(\alpha\beta_1+\beta d_1)v_1+(\alpha\beta_2+\beta d_2)v_2+\dots+(\alpha\beta_n+\beta d_n)v_n$$

$$= p_1 u_1+p_2 u_2+\dots+p_n u_n+q_1 v_1+q_2 v_2+\dots+q_n v_n$$

$$\in [X_1\cup X_2]$$

$\therefore$ $[X_1\cup X_2]$ is a subspace of V. □

2) Let $X_1=\{u_1,u_2,\dots,u_n\}$

$X_2=\{v_1,v_2,\dots,v_n\}$

Given that X_1 and X_2 are subspaces of V.

$\because$ X_1 is a subspace of V

so, $u_i+u_j\in X_1$ $\forall$ $1\le i,j\le n$

and $\alpha_i u_i\in X_1$ $\forall$ $1\le i\le n$.

$\because$ X_2 is a subspace of V

so $v_i+v_j\in X_2$ $\forall$ $1\le i,j\le n$

and $\beta_i v_i\in X_2$ $\forall$ $1\le i\le n$.

$\alpha_i u_i\in X_1$, $\beta_i v_i\in X_2$ $\forall$ $1\le i\le n$

$\Rightarrow$ $\alpha_i u_i+\beta_i v_i\in X_1+X_2$ $\forall$ $1\le i\le n$

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