• **Basis of a V.S $V$ :-**

A subset $B$ of a V.S $V$ is said to be a basis for $V$ if (i) $B$ is LI.

(ii) $[B] = V$ i.e. $B$ generates $V$.

Th

Ex:- $V = \mathbb{R}^3$

$B = \{(1,0,0), (0,1,0), (0,0,1)\}$

A:- $\alpha(1,0,0) + \beta(0,1,0) + \gamma(0,0,1) = 0$

$\Rightarrow (\alpha, \beta, \gamma) = 0$

$\Rightarrow \alpha = 0, \beta = 0, \gamma = 0$

$\therefore$ $B$ is LI.

* Now, $x_1(1,0,0) + x_2(0,1,0) + x_3(0,0,1) = (x_1, x_2, x_3)$, $x_1, x_2, x_3 \in \mathbb{R}$.

$\therefore$ $B$ generates $\mathbb{R}^3$

Let $(x,y,z) \in \mathbb{R}^3$

$(x,y,z) = \alpha(1,0,0) + \beta(0,1,0) + \gamma(0,0,1)$

$= (\alpha, \beta, \gamma)$

$\therefore \alpha = x, \beta = y, \gamma = z$.

$\therefore$ ~~B genera~~ Every elements of $\mathbb{R}^3$ can be represented as linear combination of $\{(1,0,0), (0,1,0), (0,0,1)\}$