$$\int_a^b f(x)dx$$

$$\frac{dy}{dx} = \lim_{dx \to 0} \frac{f(x+dx) - f(x)}{dx}$$

WHAT IS CALCULUS ???

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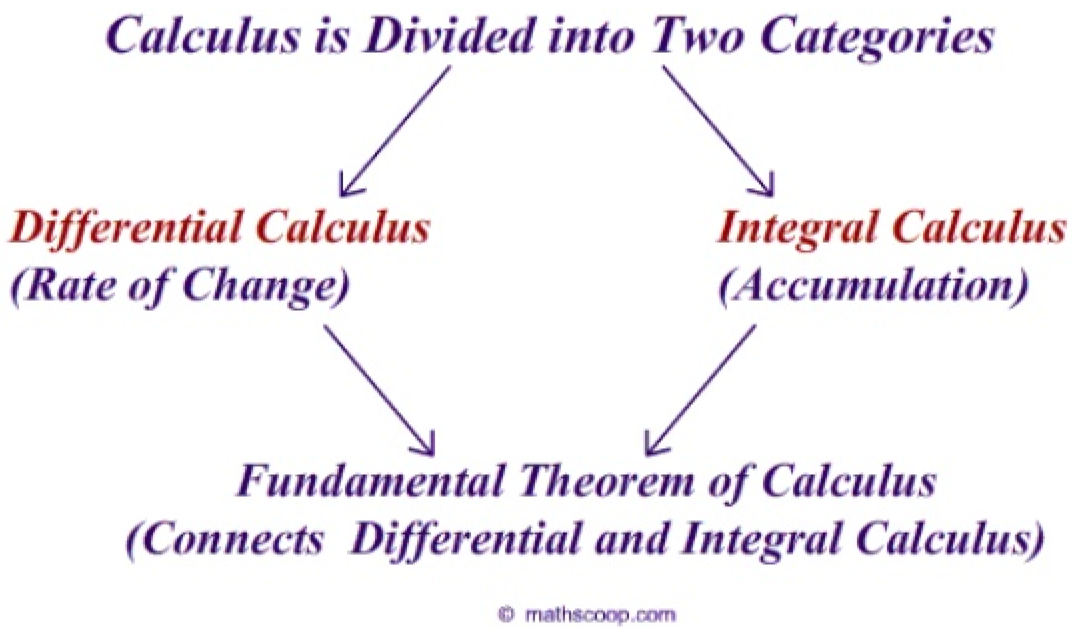
What is calculus???

Calculus is the study of change, with the basic focus being on

- Rate of change
- Accumulation



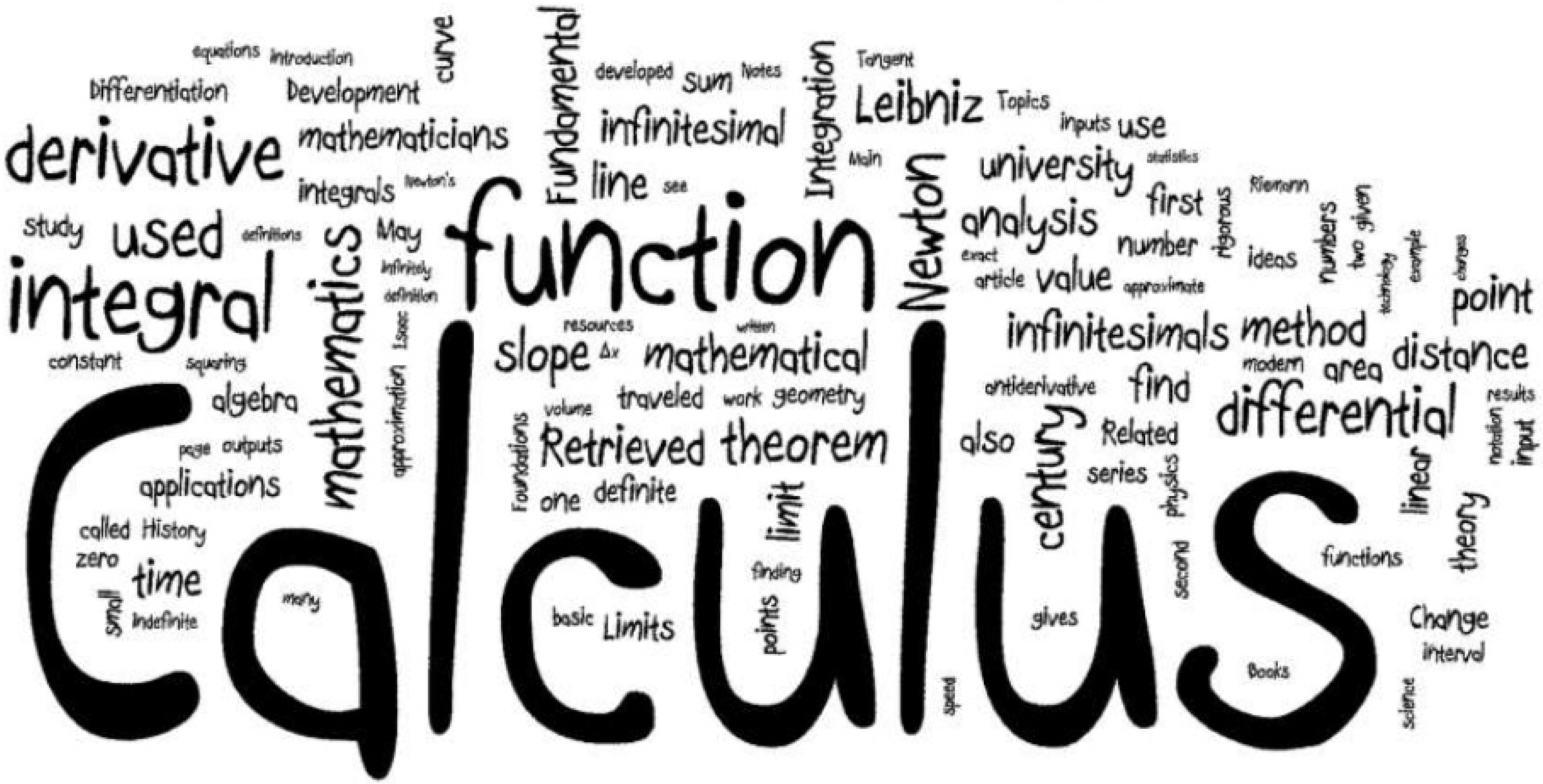
$$\frac{dy}{dx} = \lim_{dx \to 0} \frac{f(x+dx)-f(x)}{dx}$$

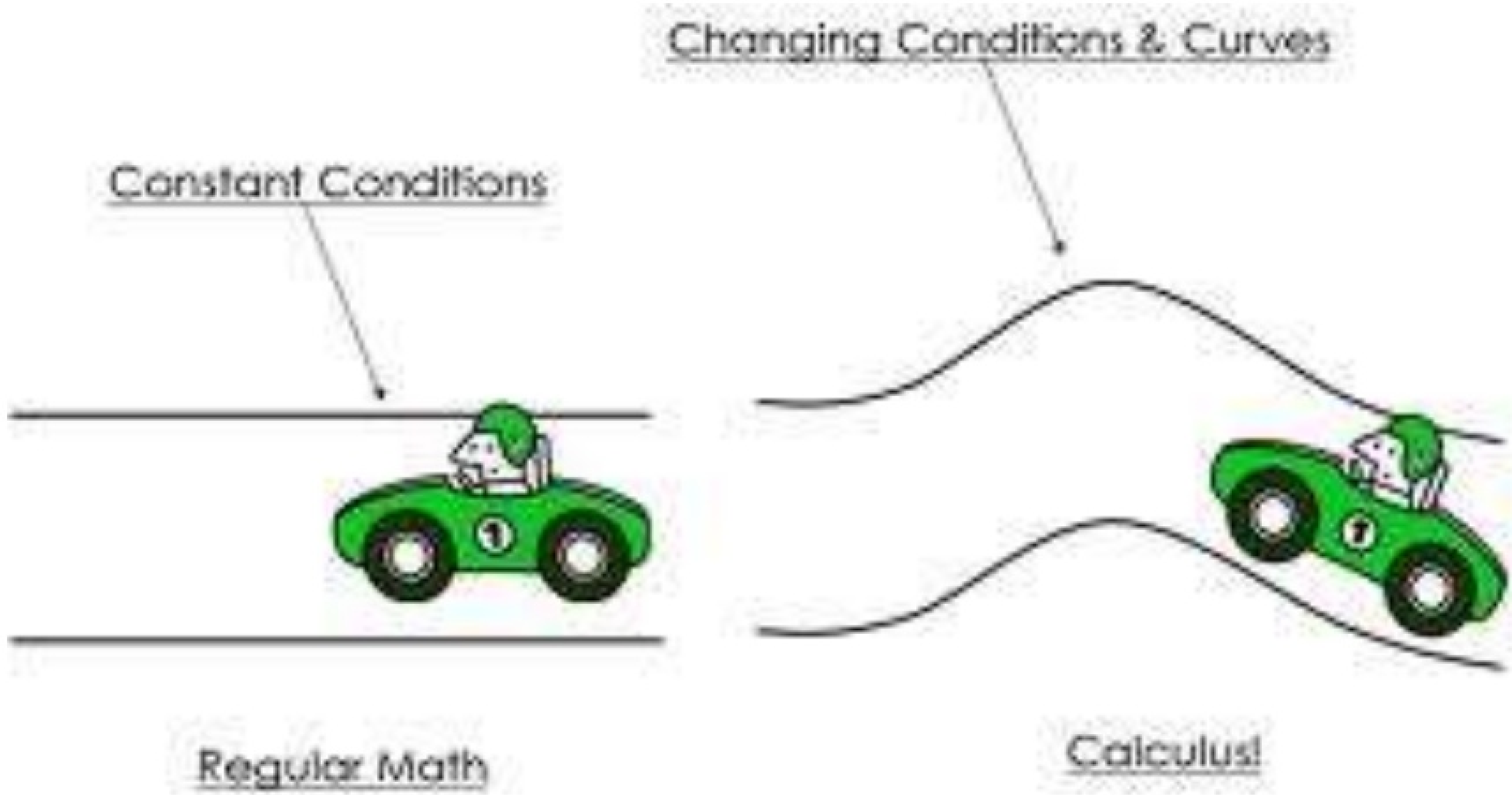
- In both of these branches (Differential and Integral), the concepts learned in algebra and geometry are extended using the idea of limits.
- Limits allow us to study what happens when points on a graph get closer and closer together until their distance is infinitesimally small (almost zero).
- Once the idea of limits is applied to our Calculus problem, the techniques used in algebra and geometry can be implemented.

Some Important Limits:

- $\lim_{x \to 0} \sin x = 0$
- $\lim_{x \to 0} \cos x = 1$
- $\lim_{x \to 0} \frac{\sin x}{x} = 1 = \lim_{x \to 0} \frac{x}{\sin x}$
- $\lim_{x \to 0} \frac{\tan x}{x} = 1 = \lim_{x \to 0} \frac{x}{\tan x}$
- $\lim_{x \to 0} \frac{\log(1+x)}{x} = 1$
- $\lim_{x \to 0} e^x = 1$
- $\lim_{x \to 0} \frac{e^x - 1}{x} = 1$
- $\lim_{x \to 0} \frac{a^x - 1}{x} = \log_e a$
- $\lim_{x \to \infty} \left(1 + \frac{1}{x}\right)^x = e$
- $\lim_{x \to \infty} \left(1 + \frac{1}{x}\right)^x = e = \lim_{x \to -\infty} \left(1 + \frac{1}{x}\right)^x$

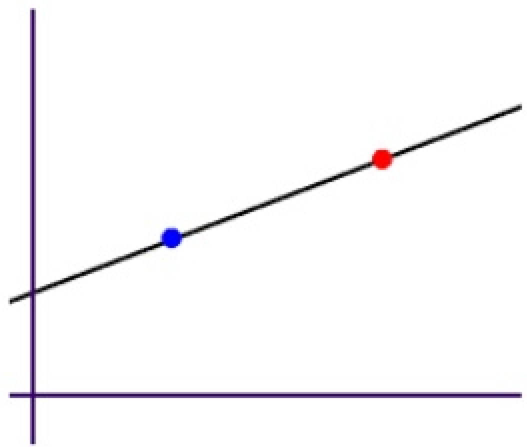
Differential Calculus $\frac{dy}{dx}$




Changing Conditions & Curves
Constant Conditions
Regular Math
Calculus!

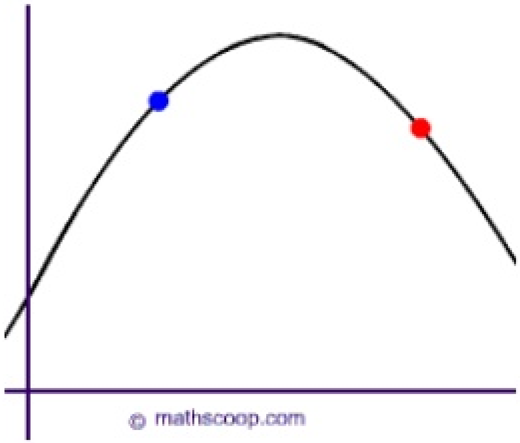
Algebra vs Calculus

In Algebra, we are interested in finding the slope of a line.

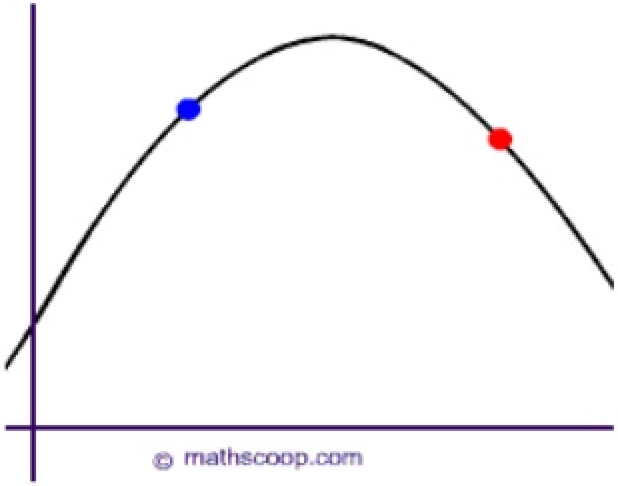


The slope of the line is the same everywhere. The slope is constant and is found using Delta y/Delta x $\frac{\Delta y}{\Delta x}$

In Calculus, we are interested in find the slope of a curve

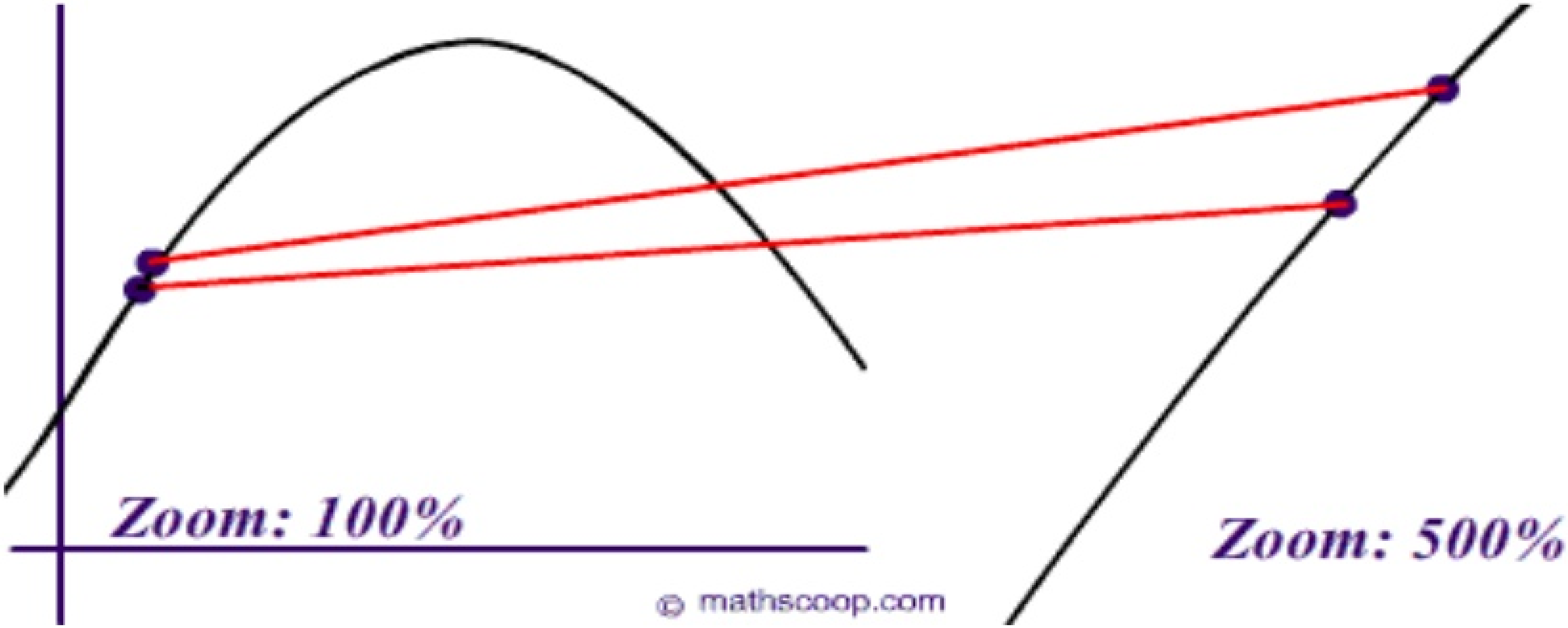


The slope varies along the curve, so the slope at the red point is different from the slope at the blue point. We need Calculus to find the slope of the curve at these specific points



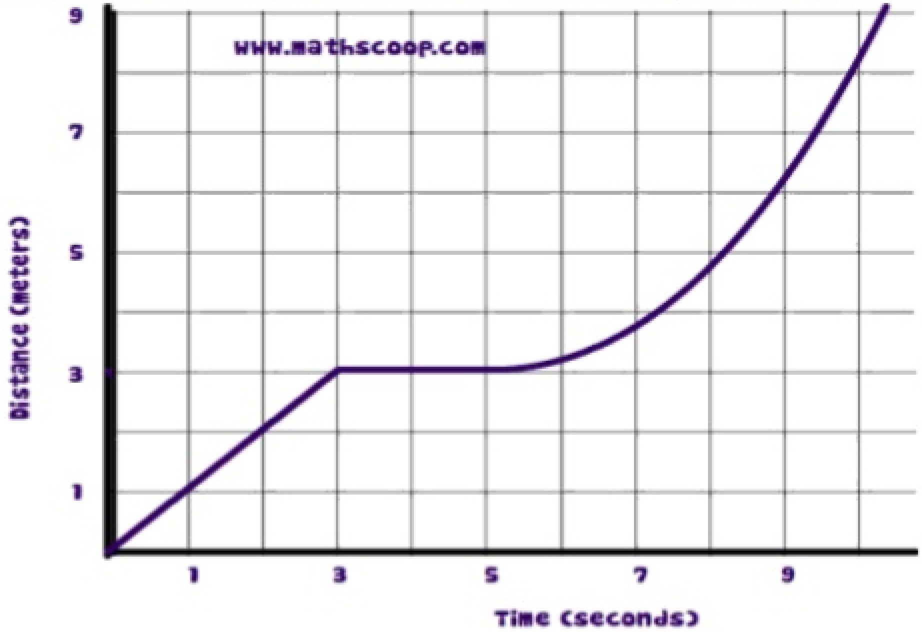
How does calculus help with curves???

- To solve the question on the Calculus side for the red point, we will use the same formula that we used in Algebra--the slope formula Delta y/Delta x
- However, we are going to make the blue and red points extremely close to each other.
- The key is, when the blue point is infinitesimally close to the red point, the curve becomes a straight line and Delta y/Delta x will then give us an accurate slope.



the more we 'zoom in' (Or the closer that the two points become), the more the curve approaches a straight line.

Differential Calculus in the Real World

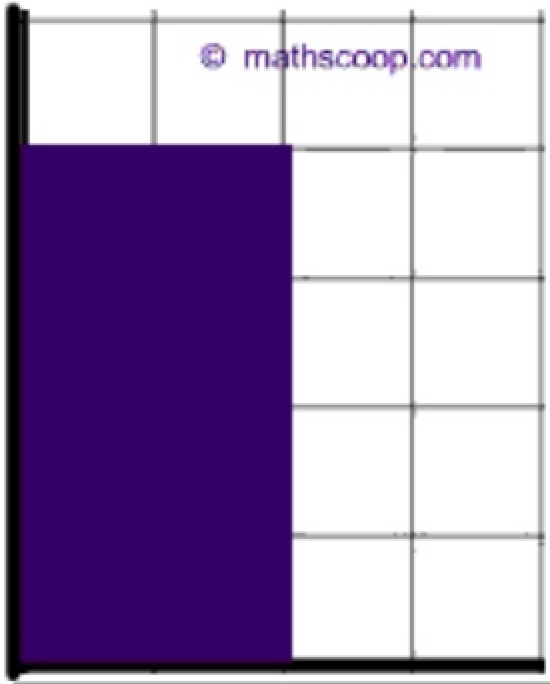


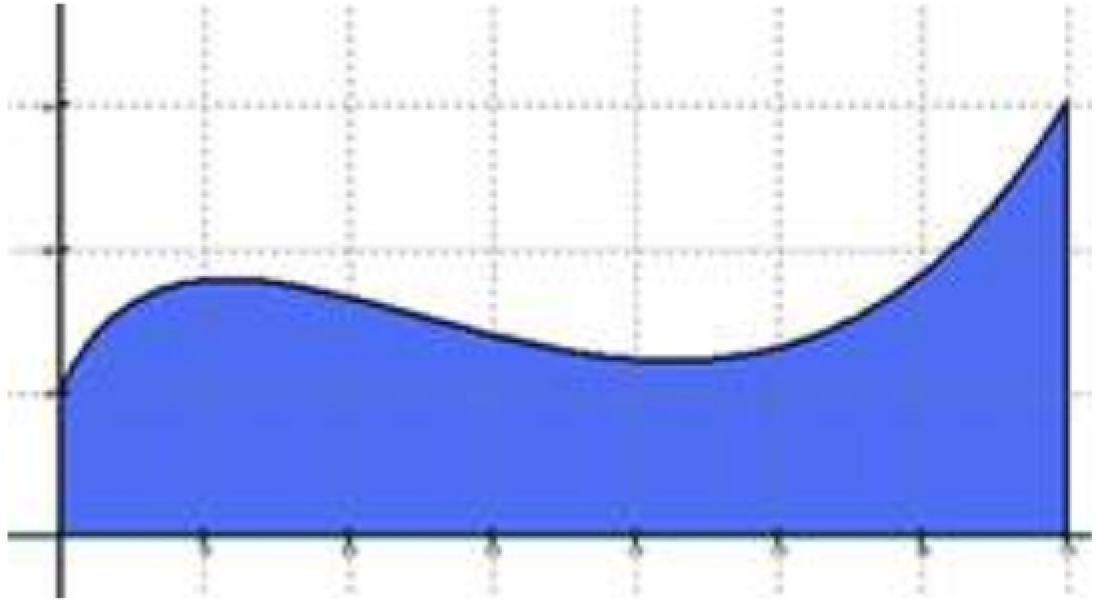
Average Velocity during the first 3 seconds?	Instantaneous Velocity at 6 seconds ?
Calculus is not needed. **velocity** **= {Delta distance}/{Delta time}** **= {3-0}{3-0}= 1**	Calculus is needed. We will need to use the method described above and try to bring two points infinitesimally close to each other.

Integral Calculus

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

Algebra vs Calculus



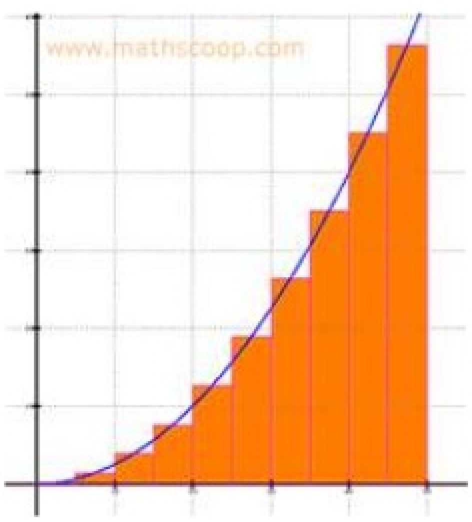


Find the area of purple region Does not require Calculus. It is simply the area of a rectangle (base)X(height). Area = 2 × 4 = 8	Find the area of blue region To find the area of blue region, we need Calculus. What can we do?

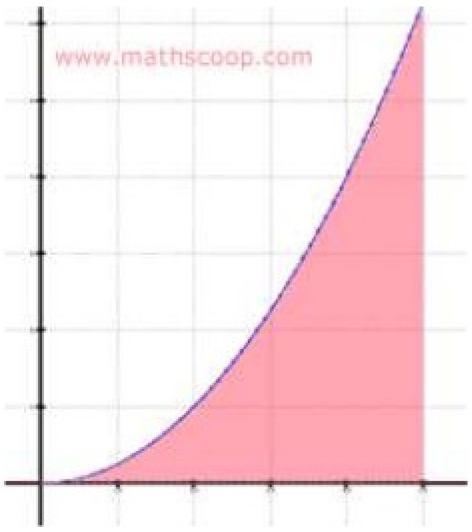
How does calculus help find the area under curves???

- Calculus lets us break up the curved blue graph into shapes whose area we can calculate--rectangles or trapezoids. We find the area of each individual rectangle and add them all up.
- The key is : the more rectangles we use, the more accurate our answer becomes. When the width of each rectangle is infinitesimally small , then our answer is precise. See the example below:

10 rectangles



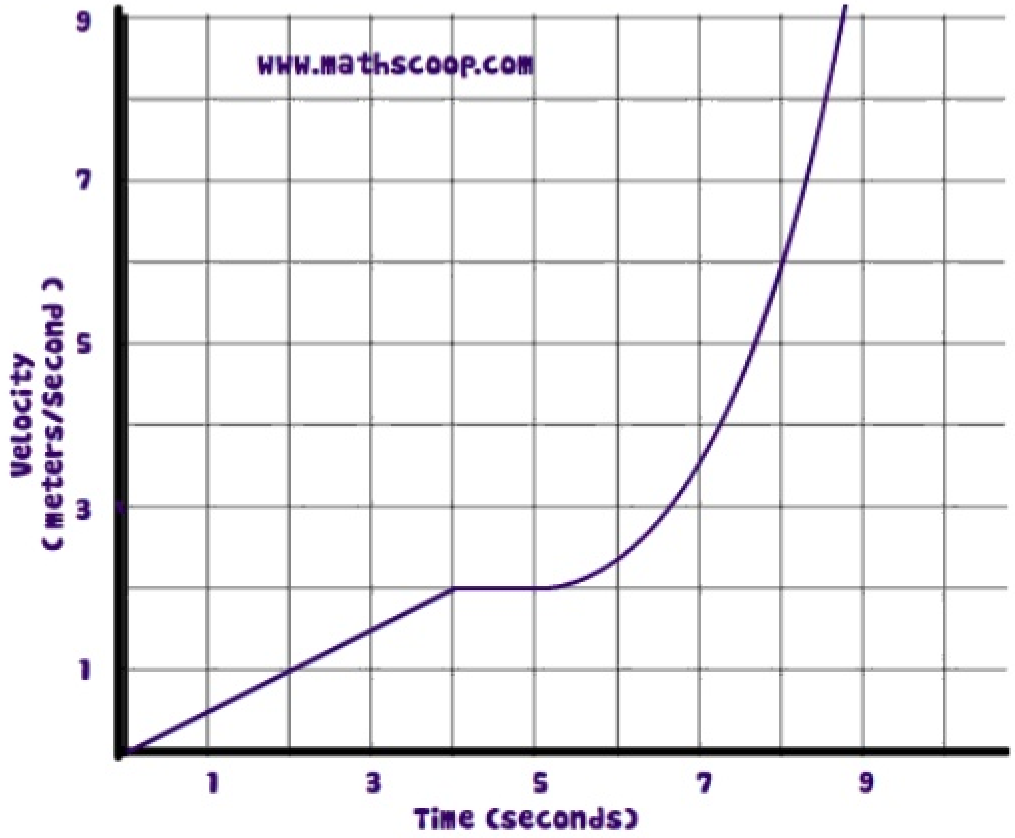
50 rectangles



Integral Calculus in the Real World

This same idea can be applied to real world situations. Consider the **velocity** vs time graph of a person riding a bike.

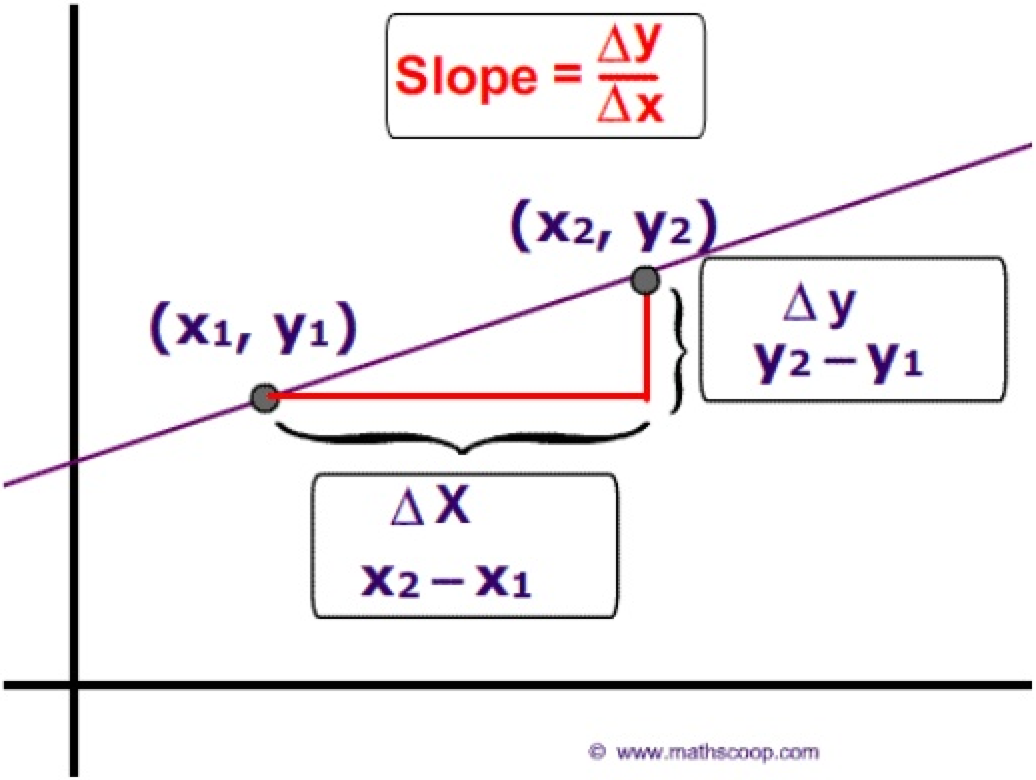
Note: this is not the same graph that we looked at above. The first one that we looked at was distance vs time



Find the distance traveled during the first 3 seconds?	Find the distance traveled during the first 9 seconds?
Calculus is not needed. Distance = (velocity)(time) This is found, by looking at area under the velocity curve bounded by the x-axis. So we just have to find the area of the triangle from x=0 to x=3.	Calculus is needed. We will need to use the method described above and find the area of infinitesimally small rectangles/trapezoids.

Slope of a line

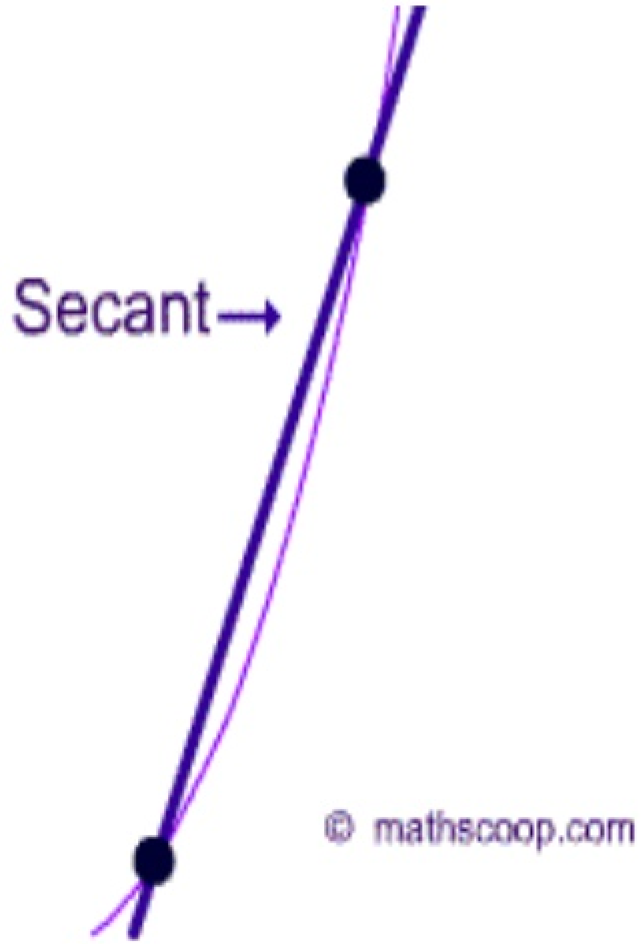
- The formal definition of a derivative is based on calculating the slope of a line.
- The typical method for finding the slope is $\frac{\Delta y}{\Delta x}$ We use any two points on the line and substitute them into the formula, as shown in the picture below.



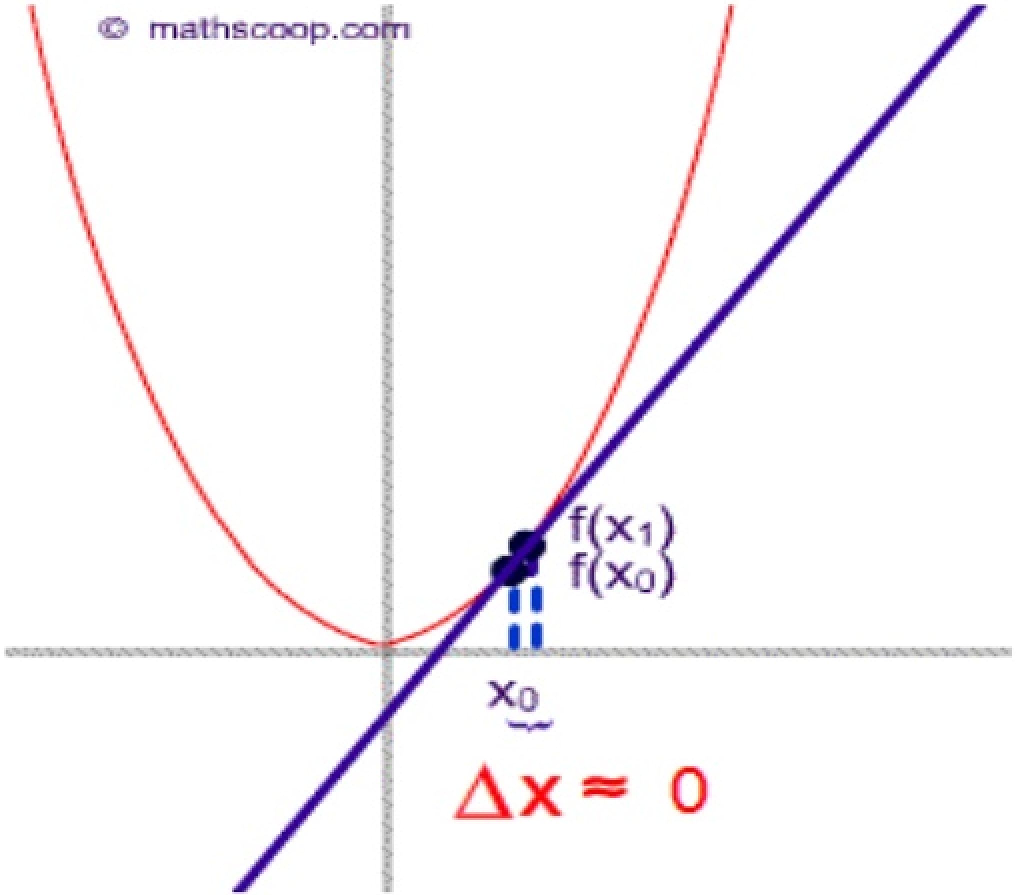
Slope of a Secant Line

- If we draw a line connecting two points on a curve this line is called a secant line. We can find its slope using the method above, . We have two points on the line, so using the formula, gives us the slope of the secant line.

- The slope of the secant line gives you the average rate of change of the curve between the two points.



as the distances approach zero, the secant line approaches the tangent.



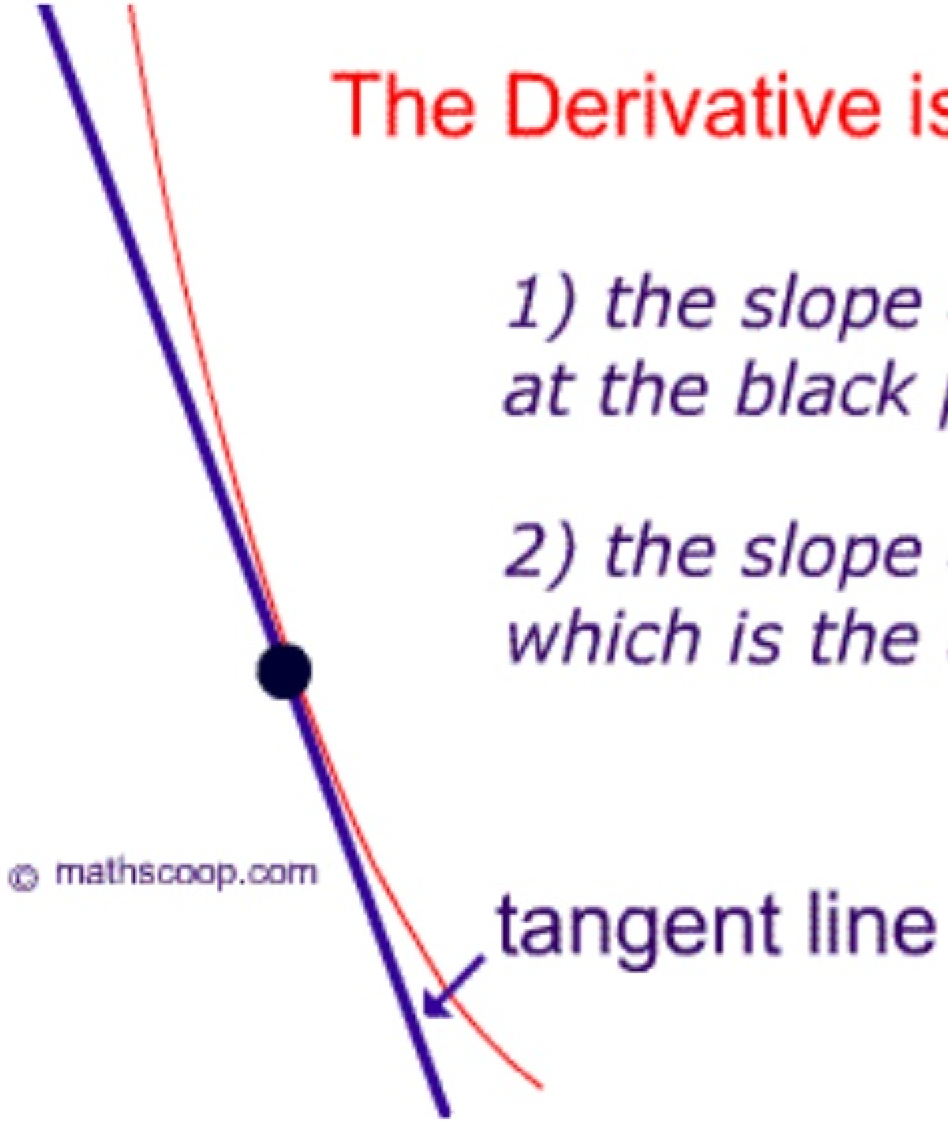
A derivative is basically the slope of a curve *at a single point*.

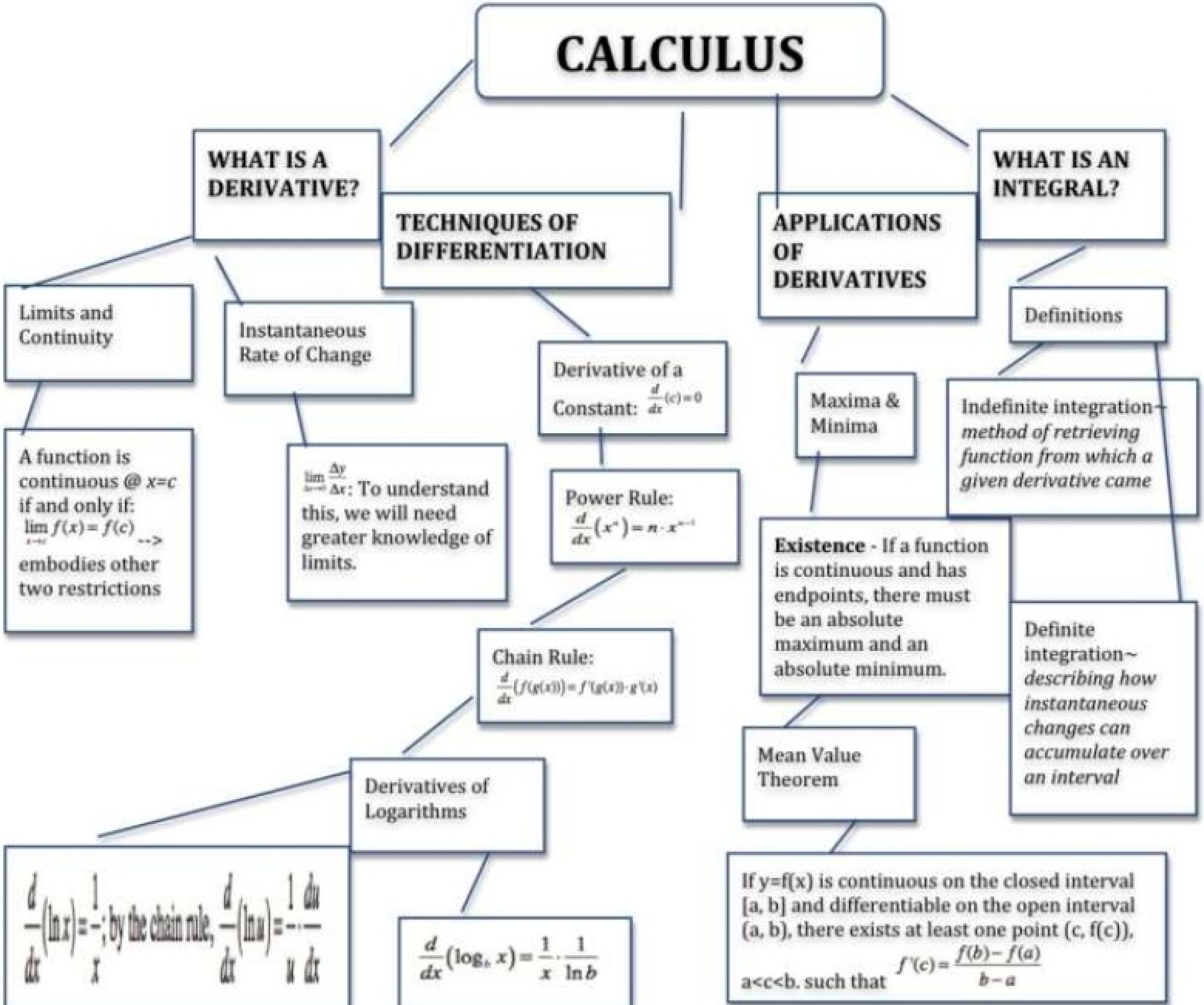
The derivative is also often referred to as *the slope of the tangent line* or *the instantaneous rate of change*.

The Derivative is

1) the slope of the red curve at the black point

2) the slope of the purple line which is the tangent line




CALCULUS
WHAT IS A DERIVATIVE?
TECHNIQUES OF DIFFERENTIATION
APPLICATIONS OF DERIVATIVES
WHAT IS AN INTEGRAL?
Limits and Continuity
Instantaneous Rate of Change
Derivative of a Constant: $\frac{d}{dx}(c)=0$
Maxima & Minima
Definitions
A function is continuous @ $x=c$ if and only if: $\lim_{x\to c} f(x)=f(c)$ --> embodies other two restrictions
$\lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x}$: To understand this, we will need greater knowledge of limits.
Power Rule: $\frac{d}{dx}(x^n)=n\cdot x^{n-1}$
Indefinite integration~ *method of retrieving function from which a given derivative came*
Existence - If a function is continuous and has endpoints, there must be an absolute maximum and an absolute minimum.
Chain Rule: $\frac{d}{dx}(f(g(x)))=f'(g(x))\cdot g'(x)$
Definite integration~ *describing how instantaneous changes can accumulate over an interval*
Mean Value Theorem
Derivatives of Logarithms
$\frac{d}{dx}(\ln x)=\frac{1}{x}$; by the chain rule, $\frac{d}{dx}(\ln u)=\frac{1}{u}\cdot\frac{du}{dx}$
$\frac{d}{dx}(\log_b x)=\frac{1}{x}\cdot\frac{1}{\ln b}$
If y=f(x) is continuous on the closed interval [a, b] and differentiable on the open interval (a, b), there exists at least one point (c, f(c)), a<c<b, such that $f'(c)=\frac{f(b)-f(a)}{b-a}$