Numerical Simulation of Flow of Bulk Solids Using CANAsand Constitutive Model

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***Abstract**: The CANAsand constitutive law in conjunction with the ID technique is used to study the flow phenomenon in a cohesionless granular medium placed between two parallel, rough vertical walls. It is shown that the development of flow is influenced by the geometry of the case. However the main factor is the void ratio of the medium: i.e. arching will prevent the free flow of the material if its void is close to the compact state. The study is extended to cover the axisymmetric situation. Here the flow of bulk solids through a circular opening at the base of a cylindrical tank is examined.*

***Keywords:** CANAsand model, granular material, bulk solid, flow property, compact state, arching*

INTRODUCTION

The phenomenon of arching, a situation that prevents the free flow of bulk solids, has been the subject of study by many researchers in the field of civil and mining engineering. Among these scholars the works of Terzaghi (1943), Krynine (1945), Lusher and Hoeg (1964) and Handy (1985) are noteworthy. All these authors use a statically admissible stress field as a starting point. In variance to these authors, Poorooshasb and Hassani (1989) presented a kinematically admissible solution. The study, however, was rather primitive as it did not take into account the geometry of the case: It assumed the backfill to be of an infinite extent in the vertical direction. In this paper this shortcoming is removed and the work is extended to cover the case of flow of solids through an opening in the base of a vertical cylindrical tank.

The present paper is written in the following sequence. First a very brief account of both the CANAsand model and ID technique are given. Then the results of several tests performed are presented and discussed; it is followed by a brief concluding remark.

CANAsand MODEL

The CANAsand constitutive law is one the oldest, yet simplest models in existence. It was first proposed in 1966 and ever since it has been revised and modified such that now it can accommodate quite complicated loading paths. A rather detailed description of this model was published recently, Poorooshasb and Noorzad (1996) and hence its representation here would be redundant. Two differences exist, however, between the model used in the present paper and the paper just cited.

The first difference is in the choice of stress parameters used to define the state of the sample. In the previous paper, following Roscoe et al. (1958) the stress parameters τ and σ where the shear stress acting along, and normal, to the plane of simple shear respectively. In the present work the two invariants of the stress tensor are used. Thus the stress parameters used to define the state

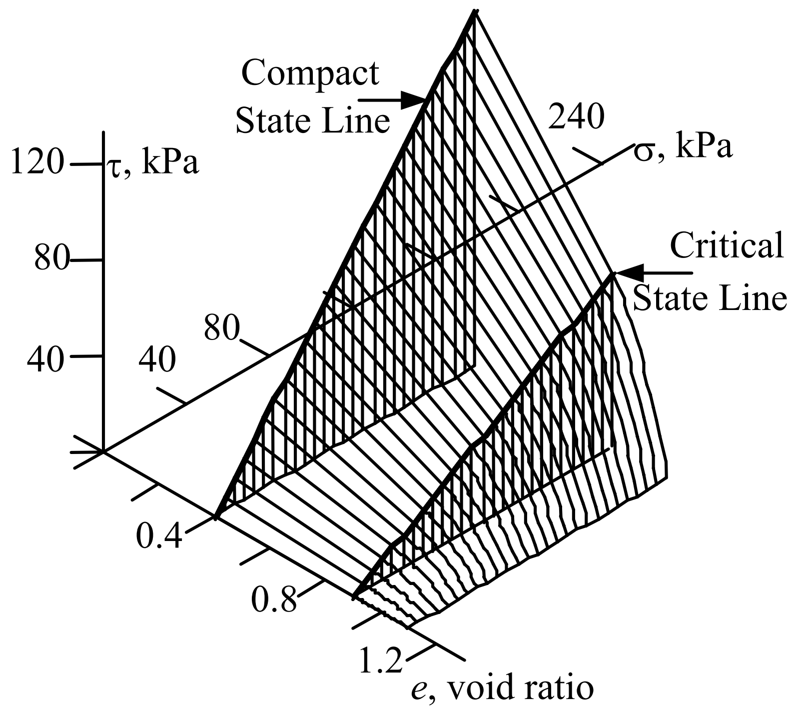

Figure 1- State Boundary Surface for a typical granular, $\phi_{critical} = 30^0; \quad \phi_{compact} = 50^0$

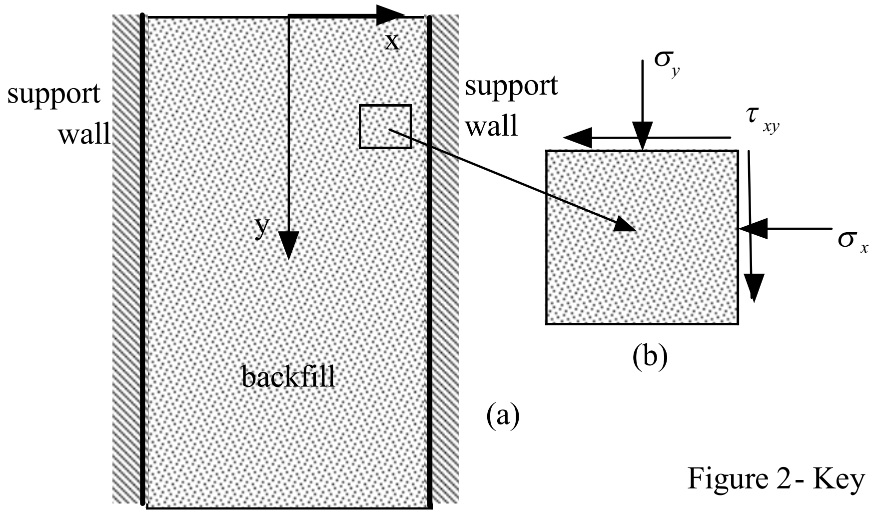

Figure 2- Key Figure

of the sample are, Poorooshasb and Noorzad (2002);

$$\sigma = (\sigma_{xx} + \sigma_{yy})/2;$$
$$\tau = [(\sigma_{xx} - \sigma_{yy})^2 / 4 + \tau_{xy}^2]^{1/2}$$

where σ_{xx}; σ_{yy}; *and* τ_{xy} are the components of the stress tensor in two dimensions.

The second difference is that in the present paper the granular is assumed to have pressure insensitive Critical and Compact void ratios. That is the material possesses a constant Critical void ratio of 0.9 and a constant Compact void ratio of 0.4. The corresponding State Boundary Surface is shown in Figure 1.

THE ID TECHNIQUE

The central issue at the ID technique is that in certain types of problems (e.g. deformation of the soil body surrounding a vertical pile) it is sufficiently accurate to assume that one of the components of the displacement can be ignored. For example in the case of the pile the radial displacement was assumed to be small and hence ignored, Poorooshasb et al. (1996).

Figure 2 shows the case of a backfill sandwiched between two vertical walls. Consider a typical element within the backfill zone as shown in (b).

In the absence of dynamic forces the incremental equation of equilibrium along the y axis requires;

$$\frac{\partial \dot{\sigma}_y}{\partial y} + \frac{\partial \dot{\tau}_{xy}}{\partial x} = 0 \tag{1}$$

which upon integration in the y direction leads to;

$$\dot{\sigma}_y + \int_0^y \frac{\partial \dot{\tau}_{xy}}{\partial x} d\xi = 0 \tag{2}$$

Let the component of the velocity field along the y axis be denoted by v. The component along the x axis by assumption is zero. The Equation (2) upon, neglecting terms of second order, reduces to;

$$c_{11}(y)\frac{\partial v(y)}{\partial y} + c_{12}(y)\frac{\partial v(y)}{\partial x} + \int_0^y [c_{21}(\xi)\frac{\partial^2 v(\xi)}{\partial x \partial \xi} + c_{22}(\xi)\frac{\partial^2 v(\xi)}{\partial x^2}]d\xi = 0 \tag{3}$$

where c_{11}, c_{12}, c_{21} *and* c_{22} are elastic-plastic moduli the magnitude of which depend on the state of the element, see Appendix to the paper. Note that if $c_{12} = c_{21} = 0$ then Equation (3) would be the same as that presented in the 1996 paper. To evaluate Equation (3) the integral sign is replaced by a sum sign Σ and it is written in its finite difference form. The graphical representation of the operation, however, is more complicated here in view of the presence of the terms c_{12} and c_{21}. Figure 3 shows the tree (graphical representation of the numerical scheme) as used in the present study.

Thus for a typical node the corresponding ID equation is, from Figure 3;

$$\sum_{i=1}^{i=node} \alpha(v_{nw} - v_{ne} + v_{se} - v_{sw}) + \sum_{i=1}^{i=node} \beta(v_e - 2v_i + v_w) + \alpha'(v_{east} - v_{west}) + \beta'(v_{south} - v_{node}) = 0 \tag{4}$$

There are three boundary conditions that must be satisfied. These are:

(1)- The y axis is a line of symmetry. Thus are the suffixes *w* (west) must be replaced by *e* (east). For example in the computer program it is sufficient to specify that at $x=0$, $nw=ne$, $sw=se$, etc.

(2)- At the vertical wall of the container the maximum shear stress that may be

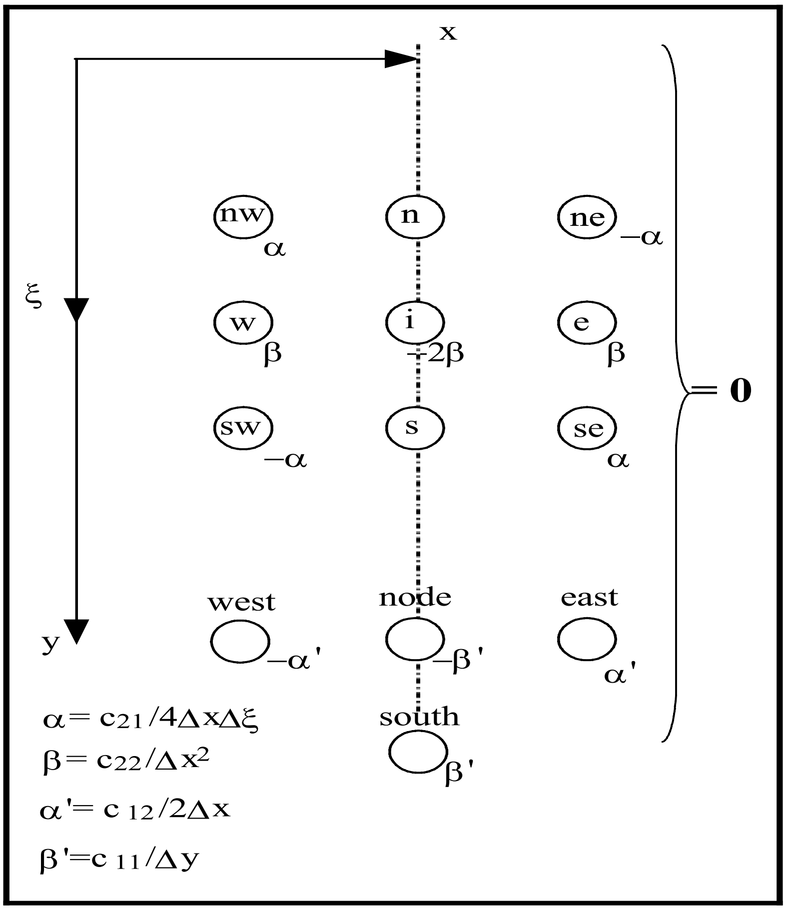

Figure 3- Graphical representation of the numerical scheme used

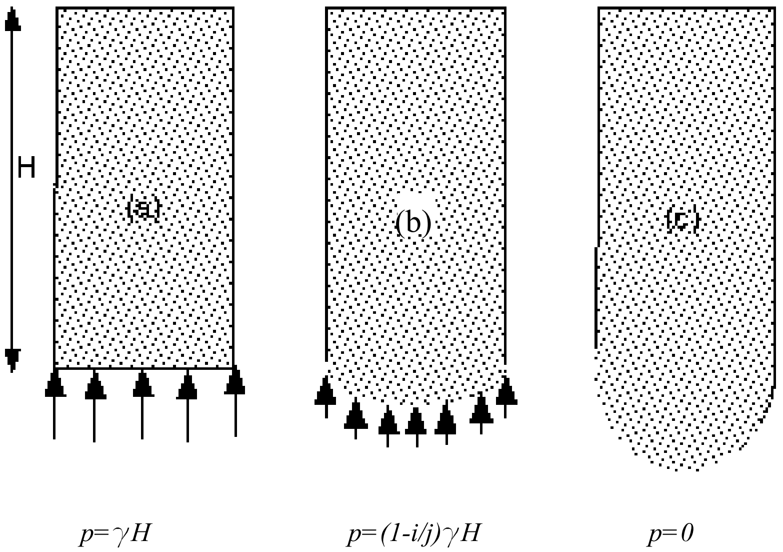

Figure 4- Boundary condition at y=H: (a) base pressure before unloading, (b) base pressure at i^{th} step, (c) zero pressure after complete removal of base

experienced by the granular is equal to $\sigma_x \, tan(\varphi_{wall})$ where φ_{wall} is the angle of friction between the wall and the granular. In this study it was assumed that, $\varphi_{wall} = 2\varphi/3$, φ being the angle of friction of the granular.

(3)- The unloading at the base was assumed to take place in a total of *j* steps. Figure 4 illustrates this operation. At step 0 (i.e., before unloading) the pressure at the base of the container is equal to γH where H is the height to which the container was filled. At the *i*th step of the unloading process the pressure at the base is $(1-\frac{i}{j})\,\gamma\, H$.

Removal of the base is indicated by zero pressure at this location.

Before leaving this section it is worth noting that;

(1)- Since the problem is non-linear the results must be checked on a continuous basis. In all evaluations reported in the following section a ten stage unloading process was used and for each stage a one-step Newton-Raphson correction method was applied.

(2)- In some instances only a portion of the base is opened representing a "door" at the base. In these cases the boundary condition at the base is only applicable to the "door" portion. For the remaining part of the base the boundary condition is simply v=0

(3)- In most cases the granular flows freely (i.e. some elements within the mass experience failure with an associated large magnitude vertical movements) before the process of unloading is complicated. In these cases arching is certainly not an issue. In some cases, however, even complete unloading does not produce free flow (i.e. only the lowest elements, at the base of the container, reach a state of failure). In these cases the development of an arch is clearly possible.

RESULTS

Three initial constant void ratios were used in all the tests the results which are presented in this section. These were *e*=0.45 a very dense material with a void ratio close to its compact state of 0.4, *e*=0.65, a medium dense media and *e*=0.85, a very loose medium having a void ratio close to its critical value of 0.9.

In the first set of examples the material is assumed to be sandwiched between two vertical walls. Such a situation may often arise in mining in the design of underground backfill operations, for example. Figure 5 shows three test results assuming a shallow depth of 1 meter and a width of 2 meters.

The "curved lines" indicate the position of originally (i.e. before unloading) horizontal lines at the moment that free flow commences. Note that the scale used to plot the vertical displacements (*u* scale) is 5 times that used to draw the geometry of the system (*x* scale).

The very dense medium (*e*=0.45) shows very little deformation nevertheless an arch can not form at such a large span. The medium dense sand (*e*=0.65) flows at the 9th step of the unloading process and is most unlikely to develop an arch. The very loose material flows at the 8th unloading stage and again it is most unlikely to form an arch.

The same comments are applicable to situations shown in Figures 6 and 7.

In Figure 6 the walls are placed at a closer

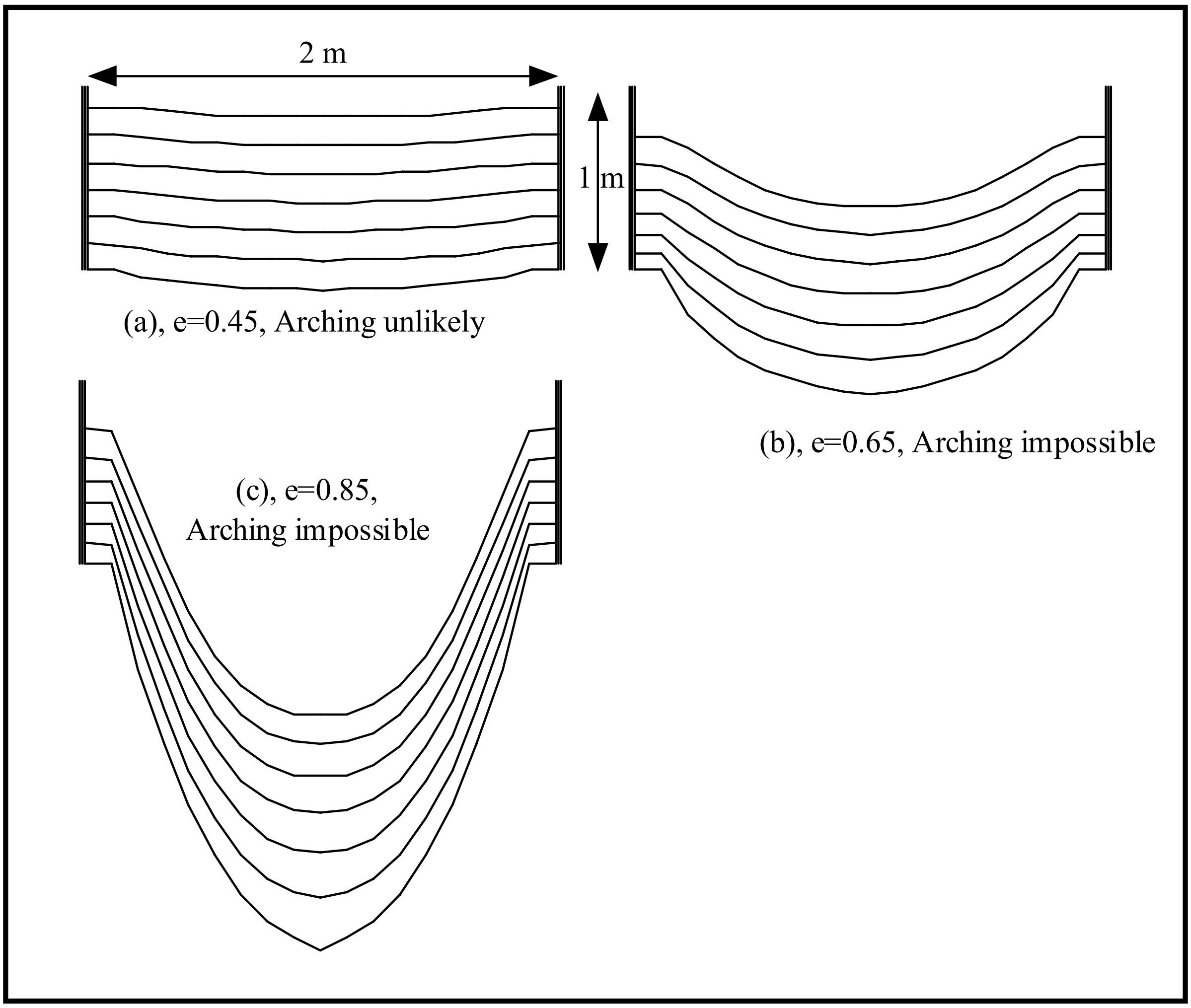

Figure 5- Flow of granular between two parallel vertical walls 2 m apart

distance of 1.0 meters while the depth is kept the same as before (1 m). In Figure 7 the walls are at the same distance but the depth of the layer is increased to 2 m.

If the flow is to take place through an opening at the base of the container then it is obvious that the chances formation arching is increased. This demonstrated in Figure 8. Note that here both very dense and the medium dense media (e=0.45 and e=0.65, respectively) survive the unloading process, i.e. arching is likely to take place. Only the very loose medium (e=0.85) flows freely at the 8th stage of the unloading process.

Two further set of tests are presented below. In these tests the container is assumed to be a vertical cylinder. Thus the problem is treated as a radial symmetry case. The corresponding ID equation is slightly more complicated since it has to accommodate an extra term representing the radial component of the stress.

Figure 9 is a counterpart to Figure 5.

Once again it is noted that the media with the higher void ratios flow freely. Arching is developed only in the very dense (e=0.45) case. Finally a situation very similar to

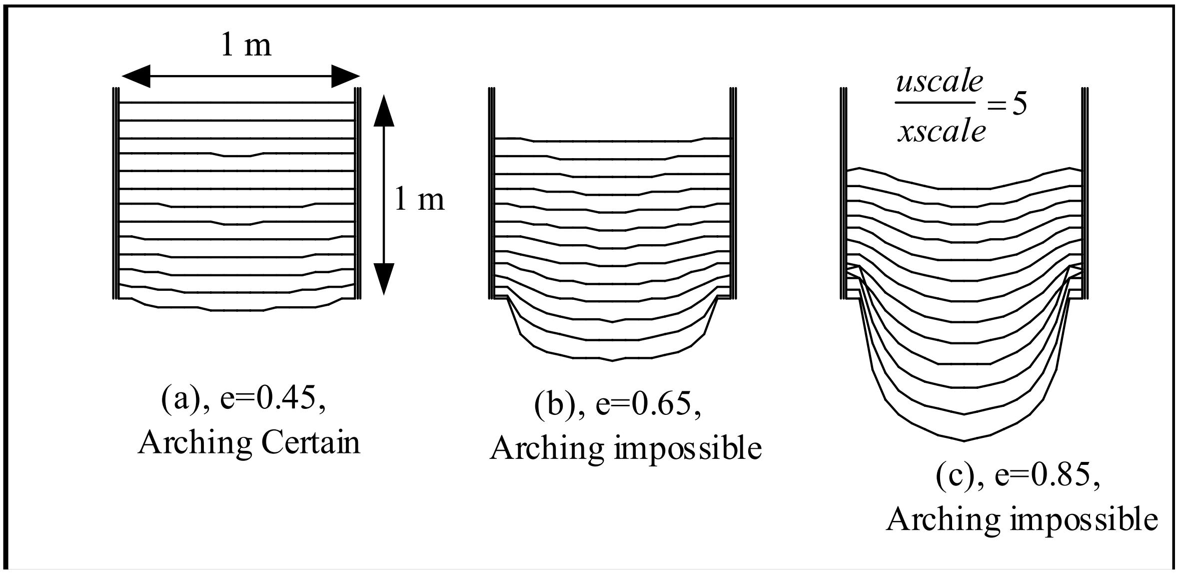

Figure 6- Flow of granular between two parallel vertical walls 1.0 m apart

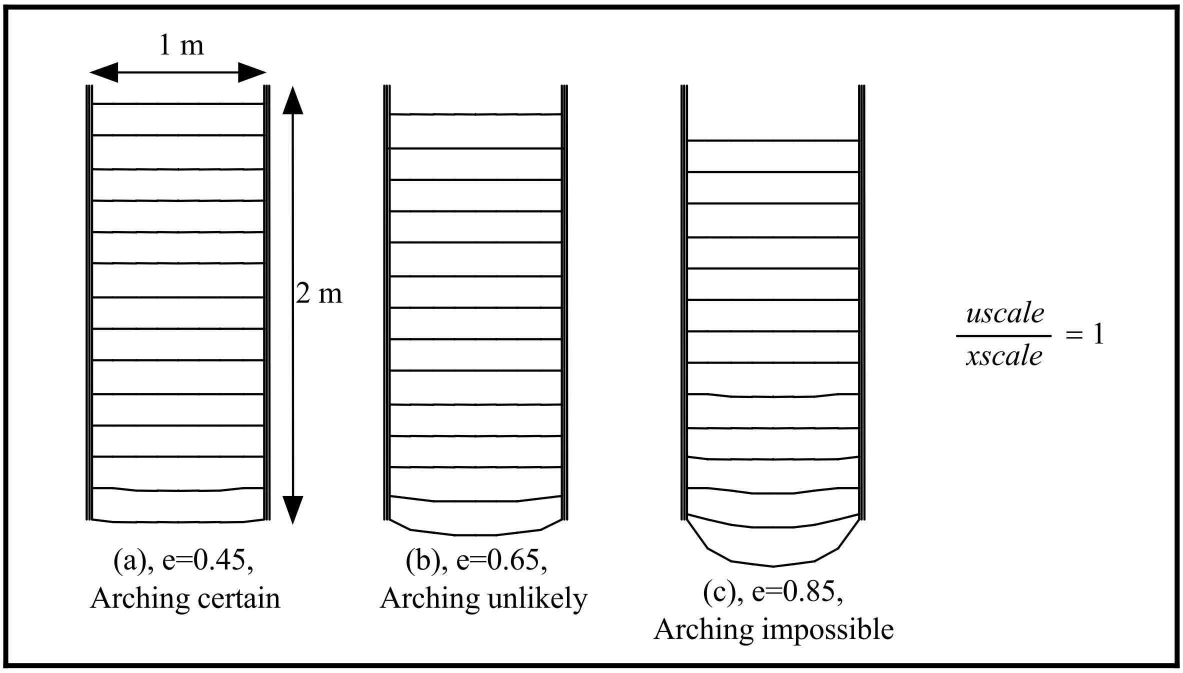

Figure 7- Flow of granular between two parallel vertical walls 1.0 m apart

Figure 6 is shown in Figure 10. Here it appears that even the very loose material may not flow freely. That is, arches may develop in all the three cases studied.

CONCLUSIONS

The CANAsand model has been used to investigate the possibility of free flow for bulk solids sandwiched between two parallel vertical walls (plane strain case) or contained in a cylindrical container.

It has been demonstrated that the most important factor which ensures free flow (prevents the formation of an arch) is the

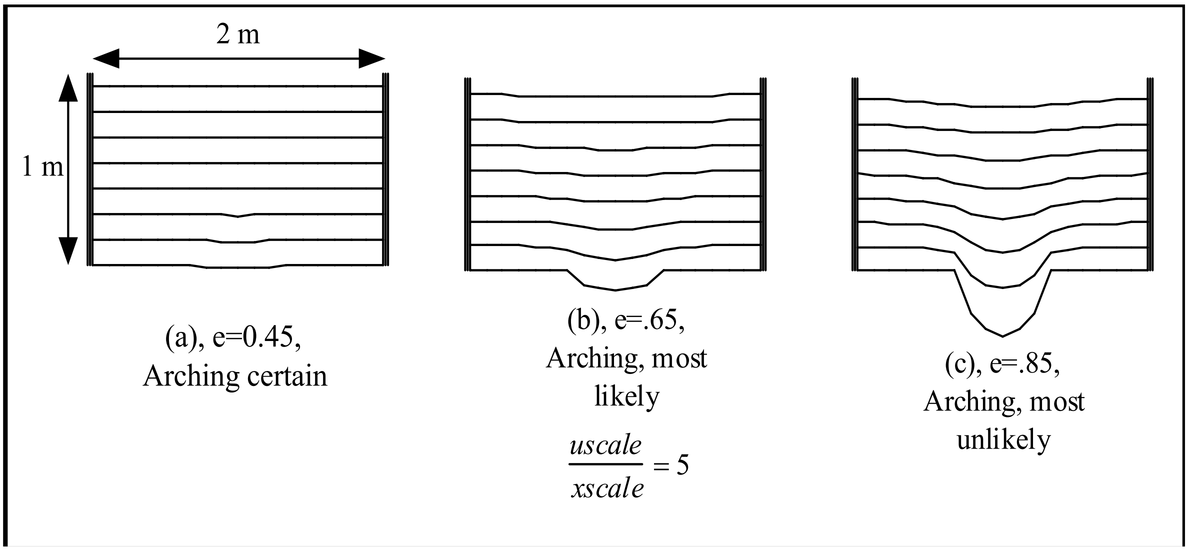

Figure 8- Flow of the granular sandwiched between two vertical walls 2.0 meters apart through a 500 mm opening

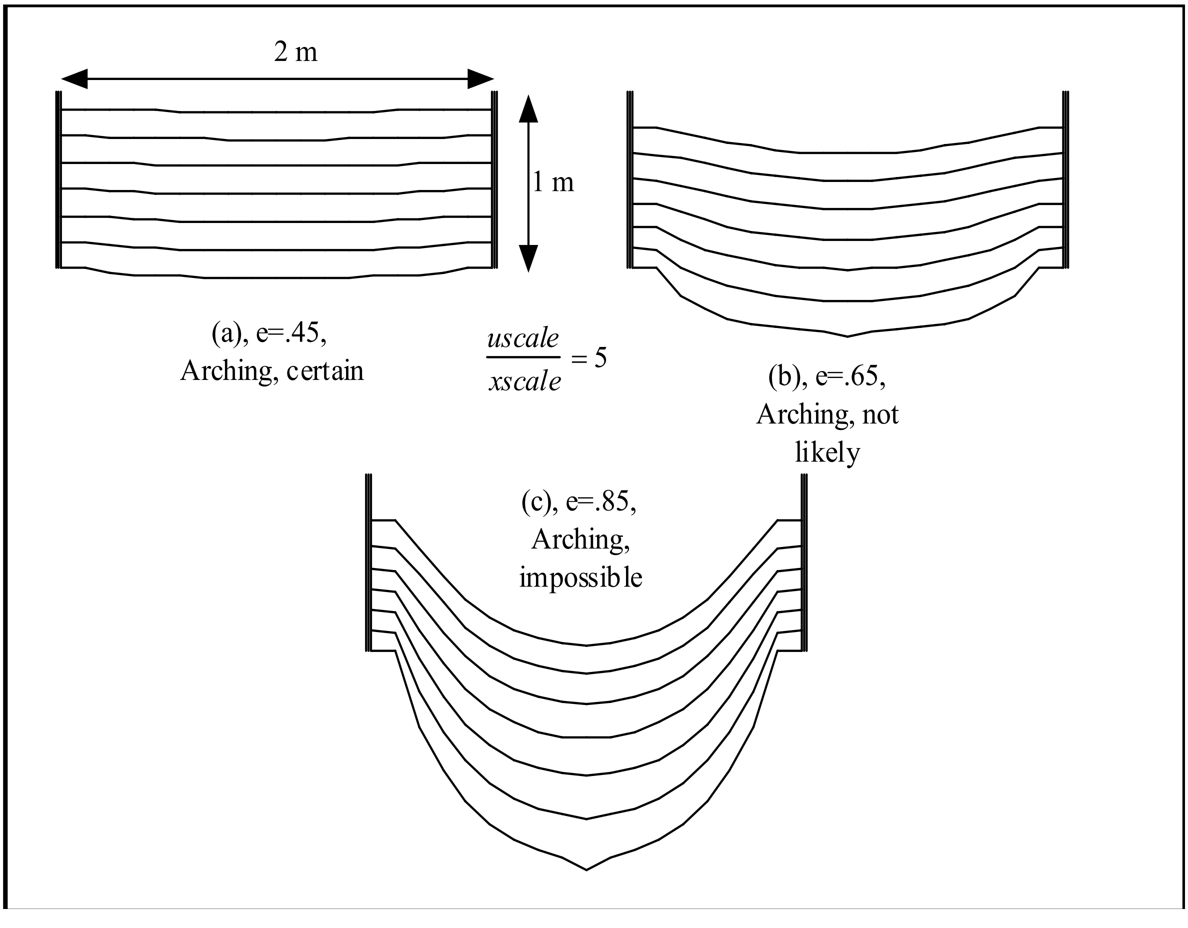

Figure 9- Flow inside a cylindrical container 2 meters in diameter

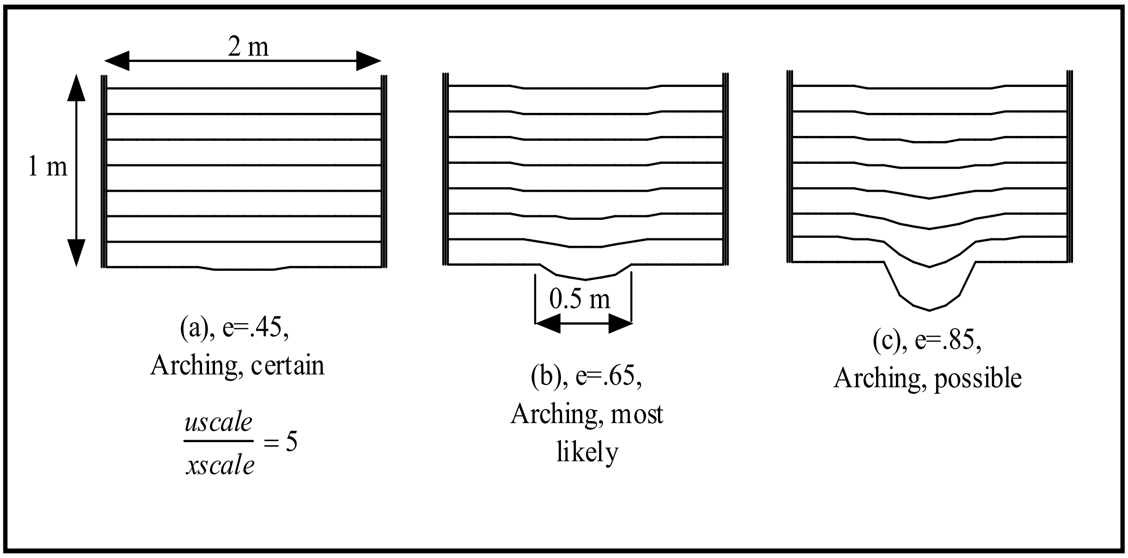

Figure 10- Flow through a 500 mm diameter opening at the base of a cylindrical container 2.0 m in diameter

void ratio of the medium stored. If the medium is very dense (void ratio close to the compact value) then arches are formed and flow will be prevented. If on the other hand the stored material is at a high void ratio (close to its critical value) then arches will not form and the material will flow freely.

The materials stored in cylindrical containers are more likely to form an arch than those stored between parallel vertical walls.

ACKNOWLEDGEMENT

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APPENDIX

The relations between the strain increment and the stress increment may be stated as;

$$\begin{bmatrix} \dot{\varepsilon}_x \\ \dot{\varepsilon}_y \\ \dot{\varepsilon}_{xy} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} \dot{\sigma}_x \\ \dot{\sigma}_y \\ \dot{\sigma}_{xy} \end{bmatrix} \qquad \text{A-1}$$

where the a_{ij} are the elastic plastic moduli. As an example the value of a_{11} is given by the equation;

$$a_{11} = h\varphi_x \eta_x + 1/E;$$
$$\varphi_x = \frac{\partial \varphi}{\partial \sigma_x} = e^{\eta/\mu}[1 + \frac{1}{\mu}(\frac{\sigma_x - \sigma_y}{\tau} - \eta)] \qquad \text{A-2}$$
$$\eta_x = \frac{\partial \eta}{\partial \sigma_x} = \frac{1}{\sigma}[\frac{\sigma_x - \sigma_y}{\tau} - \eta]$$

where σ and τ are stress invariants defined in the text and the two functions φ and η are defined by;

$$\varphi = \sigma e^{\eta/\mu}$$
$$\eta = \frac{\tau}{\sigma} = \frac{[(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2]^{1/2}}{\sigma_x + \sigma_y}$$

The expression for η is permissible since the soil is assumed to have a unique critical void ratio.

Reverting to Equation A-1 and noting that in the ID technique the lateral strain component is assumed to be zero one may obtain the equation;

$$\sigma_x = -\frac{a_{12}}{a_{11}}\sigma_y - \frac{a_{13}}{a_{11}}\tau_{xy}$$

Thus equation A-1 may be reduced to;

$$\begin{bmatrix} \dot{\varepsilon}_y \\ \dot{\varepsilon}_{xy} \end{bmatrix} = \begin{bmatrix} \lambda_{22} & \lambda_{23} \\ \lambda_{32} & \lambda_{33} \end{bmatrix} \begin{bmatrix} \dot{\sigma}_y \\ \dot{\tau}_{xy} \end{bmatrix} \qquad \text{A-3}$$

$$\lambda_{22} = a_{22} - \frac{a_{21}a_{12}}{a_{11}}; \quad \lambda_{23} = a_{23} - \frac{a_{21}a_{13}}{a_{11}};$$

$$\lambda_{32} = a_{32} - \frac{a_{31}a_{12}}{a_{11}}; \quad \lambda_{33} = a_{33} - \frac{a_{31}a_{13}}{a_{11}}$$

Inversion of the last equation leads to the equation;

$$\begin{bmatrix} \dot{\sigma}_y \\ \dot{\tau}_{xy} \end{bmatrix} = \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix} \begin{bmatrix} \dot{\varepsilon}_y \\ \dot{\varepsilon}_{xy} \end{bmatrix} \qquad \text{A-4}$$

$$c_{11} = \frac{\lambda_{33}}{\Delta}; \quad c_{12} = \frac{-\lambda_{32}}{\Delta}; \quad c_{21} = \frac{-\lambda_{23}}{\Delta};$$

$$\Delta = \lambda_{22}\lambda_{33} - \lambda_{23}\lambda_{32}$$

which is used in the text.