# How good are Portfolio Insurance Strategies?

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# Introduction and Motivation

- Increasing demand for insurance contracts which also serve as savings towards retirement
- Trade off between security of the retirement savings and participation in the market
- Solution provided to the insured:

  - Payoff of insurance linked to *underlying investment strategy*
  - *guaranteed interest rate*

- Product design: basically structured insurance products and CPPI based products

# Motivation

**Implications for risk management**

- Risk management crucially depends on the underlying investment strategy

**Perspective of insured**

- Does the insured profit from products with capital guarantee?

$\Rightarrow$ When and why are CPPI (OBPI) strategies better than OBPI (CPPI) strategies?

$\Rightarrow$ Mitigate between expected utility maximization and the comparison of stylized strategies

# Outline of the talk

- Review of the (well known) optimization problems yielding
  - constant mix,
  - CPPI
  - and OBPI strategies
- Comparison of the optimal strategies and resulting payoffs
- Discussion of some advantages (disadvantages) of the portfolio insurance methods
- Illustration of utility losses caused by the introduction of strictly positive terminal guarantees for a CRRA investor
  - effects of market frictions such as discrete–time trading, transaction costs and borrowing constraints

# Model Setup

- Assumptions

$$\begin{aligned} d\,B_t &= B_t\, r\, dt, \quad B_0 = b \\ d\,S_t &= S_t\left(\mu\, dt + \sigma\, dW_t\right), \quad S_0 = s \end{aligned}$$

  - $W = (W_t)_{0\leq t\leq T}$ standard Brownian Motion
  - $\mu$, $\sigma$ and $r$ constant ($\mu > r \geq 0$, $\sigma > 0$)

- Value Process $V = (V_t)_{0\leq t\leq T}$ of investment strategy $\pi$

$$dV_t(\pi) = V_t\left(\pi_t \frac{dS_t}{S_t} + (1-\pi_t)\frac{dB_t}{B_t}\right), \quad V_0 = A$$

# Benchmark Optimization Problems

## Optimization problems

| problem | utility function ($\gamma > 0$, $\gamma \neq 1$) | additional constraint | optimal strategy |
|---|---|---|---|
| (A) | $u_A(V_T) = \frac{V_T^{1-\gamma}}{1-\gamma}$ | none | CM |
| | unconstrained CRRA problem | | |
| (B) | $u_B(V_T) = \frac{(V_T - G_T)^{1-\gamma}}{1-\gamma}$ | none | CPPI |
| | subsistence level $G_T$ (HARA) | | |
| (C) | $u_A(V_T) = \frac{V_T^{1-\gamma}}{1-\gamma}$ | $V_T \geq G_T$ | OBPI |
| | constrained CRRA problem | | |

# Optimal Payoffs

- Optimal Payoffs

$$\begin{aligned} V^*_{T,A} &= \phi(V_0, m^*)\, S_T^{m^*} \\ V^*_{T,B} &= G_T + \alpha_B\, V^*_{T,A} \\ V^*_{T,C} &= \alpha_C\, V^*_{T,A} + \left[G_T - \alpha_C\, V^*_{T,A}\right]^+ \end{aligned}$$

- $m^* = \frac{\mu - r}{\gamma\sigma^2}$ (*Merton investment quote*)
- Fractions $\alpha_B$ and $\alpha_C$ are

$$\alpha_B = \frac{V_0 - e^{-rT} G_T}{V_0} < \alpha_C = \frac{\tilde{V}_0}{V_0} < 1$$

- Relation is also true w.r.t. more general model setups!

# Link between payoffs

- $V^*_{T,A}$ corresponds to $\phi(V_0, m^*)$ power claims with power $m^*$ where the number $\phi(V_0, m^*)$ depends on
  - the initial investment
  - and the optimal investment weight $m^*$
- Subsistence level in (B) and terminal constraint in (C) imply
  - reduction of the number of power claims (to afford the risk–free investment which is necessary to honor the guarantee)

# Link between strategies

- CPPI strategy is a
  - buy and hold strategy of a constant mix strategy
  - with an additional investment into $G_T$ zero bonds
- Solution of (C) (OBPI) is a
  - buy and hold strategy of a constant mix strategy
  - with an additional investment into a put with strike $G_T$

$\longrightarrow$ Put is cheaper than $G_T$ zero bonds such that one can buy and hold more CM strategies in the case of the option based approach

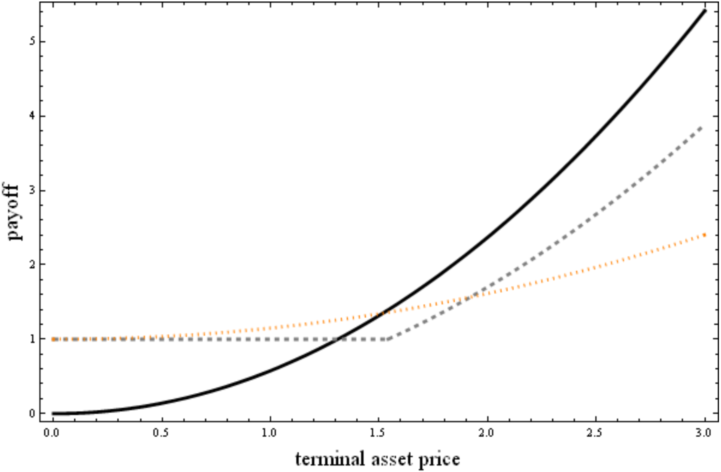
# Parameter constellation

## Basic parameter constellation

| model paramter | strategy parameter | terminal guarantee |
|---|---|---|
| $S_0 = 1$ | $V_0 = 1$ | $G_T = 1$ |
| $\sigma = 0.15$ | $T = 10$ | |
| $r = 0.03$ | $\gamma = 1.2$ | |
| $\mu = 0.085$ | $m = m^* = 2.037$ | |

Table: Basic parameter constellation.

# *Optimal* payoffs $V^*_{T,A}$ (solid line), $V^*_{T,B}$ (dotted line) and $V^*_{T,C}$ (dashed line)



# Remarks

- Intersection points with unconstrained solution
- Probability to end up with (only) the guarantee

  - OBPI payoff is equal to guarantee if the put expires out of the money
  - In contrast to the CPPI method, this implies a positive point mass for the event that the terminal value is equal to the guarantee

  $\longrightarrow$ This can cause a high exposure to gap risk, i.e. the risk that the guarantee is violated, if market frictions are introduced.

- Loss from introducing the guarantee into the unconstrained setup

# Loss rate

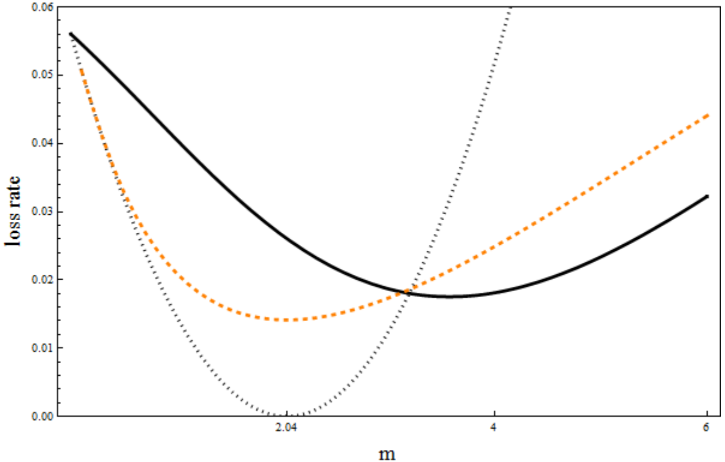
- Loss rate $l_{T,i}(\pi)$ of the strategy $\pi$ and the utility function $i$ $(i \in \{A, B, C\})$

$$
\boxed{l_{T,i}(\pi) := \frac{\ln\left(\frac{CE^*_{T,i}}{CE_{T,i}(\pi)}\right)}{T}}
$$

where

- $CE^*_{T,i}$ denotes the certainty equivalent of the optimal strategy $\pi^*_i = \left(\pi^*_{t,i}\right)_{0 \leq t \leq T}$
- $CE_{T,i}(\pi)$ the of the suboptimal strategy $\pi = (\pi_t)_{0 \leq t \leq T}$

# Loss rates w.r.t. $u = u_A$ for CPPI (solid lines), OBPI (dashed) and CM (dotted) strategies with varying parameter $m$



# Minimal loss rates ($u_A$–optimal strategy parameter $m$)

**Minimal loss rates**

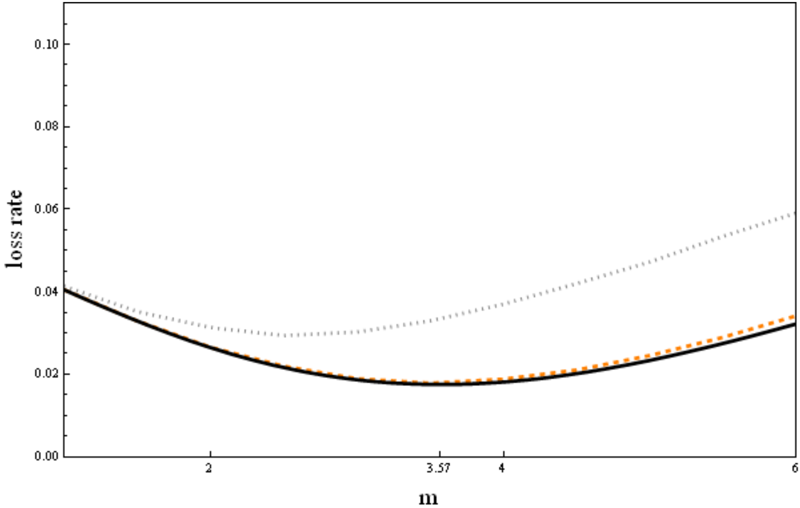
| strategy | $\gamma \setminus T$ | 1 | 2 | 5 | 10 | 2 |
|---|---|---|---|---|---|---|
| CPPI | 1.2 | 0.040 (11.32) | 0.035 (7.83) | 0.026 (4.91) | 0.018 (3.57) | 0. |
| OBPI | 1.2 | 0.037 (2.04) | 0.031 (2.04) | 0.022 (2.04) | 0.014 (2.04) | 0. |
| CPPI | 1.5 | 0.031 (10.60) | 0.026 (7.25) | 0.019 (4.45) | 0.013 (3.16) | 0. |
| OBPI | 1.5 | 0.028 (1.63) | 0.023 (1.63) | 0.015 (1.63) | 0.009 (1.63) | 0. |
| CPPI | 1.8 | 0.024 (10.03) | 0.020 (6.80) | 0.014 (4.10) | 0.009 (2.86) | 0. |
| OBPI | 1.8 | 0.021 (1.34) | 0.017 (1.34) | 0.011 (1.34) | 0.007 (1.34) | 0. |

Table: Minimal loss rates ($u_A$–optimal strategy parameter $m$) for varying $T$ and $\gamma$ where the other parameters are given as in Table 1.

- Concept of portfolio insurance is impeded by market frictions
- Asset exposure is reduced when the asset price decreases
- A sudden drop in the asset price where the investor is not able to adjust his portfolio adequately, causes a gap risk, i.e. the risk that the terminal guarantee is not achieved.
- Example: trading restrictions in the sense of discrete–time trading and transaction costs

> - Utility Loss
> - Gap risk measured by the shortfall probability

## Loss rates: continuous–time CPPI (solid line), monthly CPPI without transaction costs (dashed lines) and monthly CPPI with $\theta = 0.01$ (dotted line)



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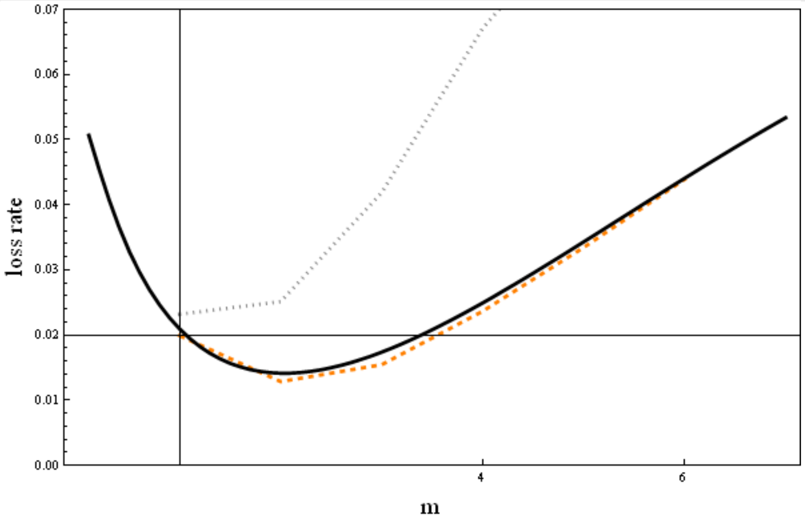
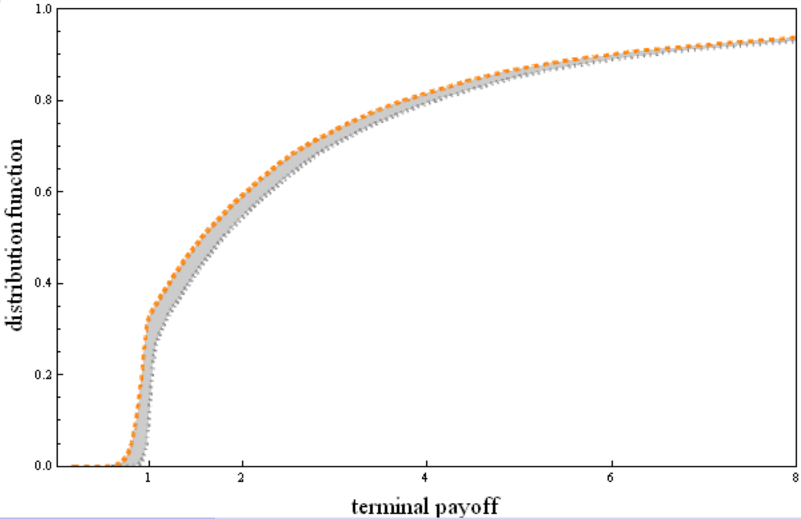


# Distribution of discrete OBPI (CPPI) with transaction costs



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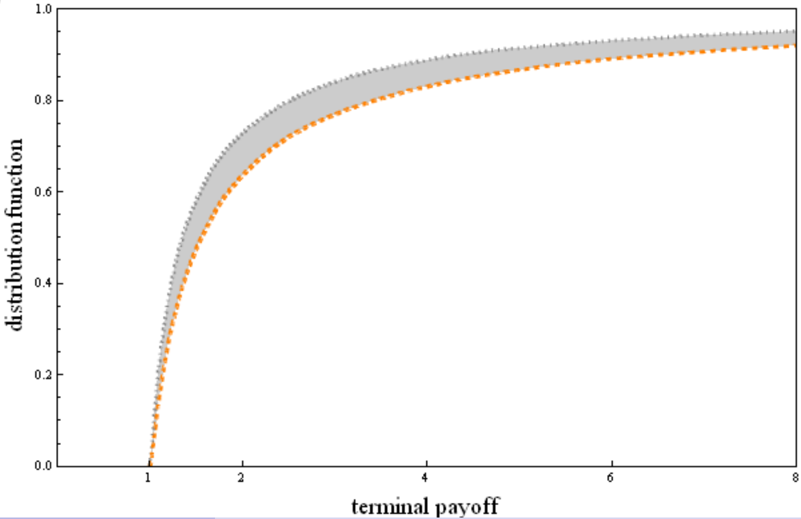

# Conclusion

- Main difference between OBPI and CPPI can be explained by their link to constant mix strategies
- It is also important to take into account the difference between kinked and smooth payoff–profiles
- Advantage of OBPI:
  - Backing up the guarantee is cheaper than for CPPI (closer to unconstrained optimal)
- Drawback of OBPI:
  - Implementation is much more difficult than the one of CPPI
  - Resulting strategies are sensitive against model risk and various sources of market incompleteness

# Thank you for your attention!