Are Referendums and Parliamentary Elections Reconcilable?
The Implications of Three Voting Paradoxes

Suzanne Andrea Bloks[*]

Abstract

In representative democracies, referendum voting and parliamentary elections provide two fundamentally different methods for determining the majority opinion. We use three mathematical paradoxes – so-called *majority voting paradoxes* – to show that referendum voting can reverse the outcome of a parliamentary election, even if the same group of voters have expressed the same preferences on the issues considered in the referendums and the parliamentary election. This insight about the systemic contrarieties between referendum voting and parliamentary elections sheds a new light on the debate about the supplementary value of referendums in representative democracies. Using this insight, we will suggest legal conditions for the implementation of referendums in representative democracies that can pre-empt the conflict between the two methods for determining the majority opinion.

Keywords: Democracy, Referendum, Elections, Majority Voting Paradoxes, Social Choice Theory

1. Introduction

The legitimacy of public decisions is not self-evident in Western democracies. This has been shown, for instance, by such instances as the decision to remain in or leave the EU, the decision to establish a political and economic association with Ukraine and the decision to accept EU migrant quotas: they have all caused public discontent. Traditionally, these decisions would be taken by representatives who are elected in the parliamentary elections of the representative democratic system, but the call for direct influence by citizens through referendums is becoming ever louder.[1]

The role of referendums in the representative system currently puzzles many minds. It is debated whether this instrument of direct democracy could and should supplement the indirect system of representative democracies. What the systems share is that they both seek a majority opinion. A (qualified) majority outcome of the votes of representatives or citizens is seen as providing legitimacy.

In this paper, we will use mathematical paradoxes to show that the search for the majority opinion in referendums is fundamentally different from such a search in the representative system. We will

---

[1] The three issues mentioned have been the subject of referendums. The discussion in the United Kingdom about exiting the EU led to the Brexit referendum in June 2016, which has disrupted British politics. The decision to establish a political and economic association with Ukraine led to a referendum in the Netherlands in April 2016, which compelled the Dutch government to negotiate additional clauses in the EU-Ukraine Agreement before ratifying it. Lastly, the decision to accept the migrant quotas imposed by the EU was the subject of a referendum in Hungary in October 2016. This referendum was much despised by the representatives of the EU and other EU member states. Although an overwhelming majority of voters rejected the decision to accept EU migrant quotas, the referendum was not binding because turnout figures did not reach the turnout quorum of 50%.

discuss three so-called *majority voting paradoxes*: the *Ostrogorski paradox*,[2] the *Anscombe paradox*[3] and *Nurmi's representation paradox*[4]. These paradoxes originate in the field of study called social choice theory, which can be regarded as a subfield of mathematics and economics.[5] They pertain to how issues are presented to voters and focus on the merits of direct *vs.* representative democracy (Nurmi 1999, p.70).[6]

Specifically, the three majority voting paradoxes elucidate the contingent features of the direct system of referendum voting and the indirect system of parliamentary elections. They show that stepwise majority formation on full policy programmes (as in parliamentary elections that include coalition formation)[7] can deliver results contrary to original majority preferences on separate issues (as voters might express in referendums). This is true even if the preferences of the individuals concerned do not change and all involved act perfectly rationally. As this is a somewhat unexpected and counter-intuitive result, they are called 'paradoxes'.

We will use this knowledge about the systemic differences between referendum voting and parliamentary elections to enhance the debate about the supplementary value of referendums in representative democracies. Some authors argue that the debate about the supplementary value of referendums revolves around a question of belief, because they think it impossible to weigh objectively the pros and cons and to determine the consequences of referendums for democratic values, such as citizen participation, the legitimacy of decision-making and the representative system (e.g. Voermans 2011, p. 469, but also Schutgens 2017, p. 141). However, knowledge about the systemic contrarieties between referendum voting and parliamentary elections can help to underpin these beliefs with knowledge.

In the legal-political debate, there is a risk that the contingent systemic features of referendum voting and parliamentary elections are overlooked when the impact of referendums in the parliamentary system is studied on a case-by-case basis. It is, therefore, striking that the three majority voting paradoxes are presented in just a small part of the literature for a select club of mathematicians and political scientists,[8] even though they were already discovered in the 1970s.

---

[2] The paradox cannot be found in any of Ostrogorski's works, but it is arguable that the author at least hinted at it in his two-volume treatise – see Ostrogorksi (1902). Rae and Daudt (1976) were the first authors to present the paradox as a dilemma between two equivalently desirable procedures (direct and representative democracy) and coined the term 'Ostrogorski paradox'.

[3] See Anscombe (1976).

[4] See Nurmi (1999, p. 78).

[5] For an introduction into social choice theory, see Arrow et al. (2002; 2011). Relevant parts about voting theory include parts 1 to 3 in the 2002 volume and part 8 in the 2011 volume.

[6] Another well-known majority voting paradox is the Paradox of Multiple Elections, analysed and introduced by Brams, Kilgour and Zwicker (1998). Nurmi (1999, pp. 74f) shows how this paradox can be deduced by changing the Ostrogorski paradox. This paradox, however, does not concern the relation between referendum voting and parliamentary voting.

[7] The models concern parliamentary elections in the broad sense of the term. Parliamentary elections in the narrow sense of the term, which refers to the voting process, are not typically concerned with establishing the majority opinion. Only in the broad sense of the term, when the notion also includes coalition formation and the hammering out of a government policy programme, are parliamentary elections aimed at establishing a policy programme that is preferred by an overall majority.

[8] For classical political science literature in which mathematical models are applied to debates in political theory, see Riker (1982) and Pettit (2012). Riker builds his argument on the *paradox of voting* and Pettit uses the *discursive dilemma.* Furthermore, for the first and (to our knowledge) only application of voting paradoxes

In the first part of this paper, we will bring out again the three majority voting paradoxes, give them a good dusting off and analyse their implications for combining referendums and parliamentary elections. This analysis should make constitutional lawyers and politicians aware of the robust systemic differences between referendum voting and parliamentary elections, and enable them to ground their beliefs on knowledge about the effects of referendums in representative democracies on the legitimacy of public decisions and the power of citizens and representatives over decision-making.

In the second part of the paper, we will take the insights gained from the three paradoxes one step further: we will examine under what conditions the negative effects of the robust systemic differences between referendum voting and parliamentary elections can be prevented from arising. We will discuss the following conditions: (1) stipulating that referendums be *legally binding*, (2) using a *sequence of referendums* on the same issue, (3) introducing a *quorum requirement* in referendums and (4) *avoiding connections* between the referendum issues and the issues considered in parliamentary elections. This discussion should enhance an understanding of the effects of imposing such conditions on referendums and will lead to recommendations for the legislature if representative democracies are to be supplemented with referendums.

2. Three Voting Paradoxes

Referendums and parliamentary elections are based on different methods for determining the will of the voters.

Referendums are based on a direct method. Voters are given the opportunity to directly express their opinion on a specific issue; they cast a *disaggregated* vote. The majority opinion is determined per issue and, in the hypothetical case that all important issues are voted on in referendums, the resulting policy programme is constituted by the policies that have been favoured *issue-wise* in the referendums.[9] We will say that the resulting policy programme or majority outcome is the joint outcome of the *disaggregated* votes.

By contrast, parliamentary elections are based on an indirect method. Voters cast their vote on a full policy programme, whereby they indirectly express their opinion on the specific issues. Voters add up their opinions on separate issues in a single parliamentary vote for a (party) candidate with a policy programme that reflects their opinions on most – or the most important – issues. The parliamentary

---

to the debate about referendums, see Nurmi (1998). Nurmi describes five paradoxes and touches on their implications. We will build on this paper by focussing on the three *majority voting paradoxes* and their implications for the legal-political debate about the supplementary value of referendums, by making a more in-depth analysis of the practical and legal implications, and by introducing new legal solutions.

[9] The voters in referendums on European cooperation, like the Brexit and Ukraine referendums, are assumed to opt for the choice or the (party) candidate that has a similar opinion on the topic. Behaviour in a referendum is thus not dissimilar from voting in parliamentary elections where voters choose the policy programme or (party) candidate that agrees with their opinions on most or the most important issues. However, a prominent alternative theory of voting behaviour in EU referendums is the 'second-order' theory of elections. In referendums as 'second-order elections', national issues tend to dominate the referendum campaign and hence voters do not express their opinion on the specific issue in the referendum but 'use their vote as a means of signalling their satisfaction or dissatisfaction with the government or to follow the recommendations of national parties.' If the latter theory of voting behaviour applies, the distinction between the disaggregated and aggregated vote is not as clear-cut as described, because the referendum vote constitutes a second-order parliamentary vote (Hobolt 2006, p. 155).

vote can also be called an *aggregated* vote, because voters combine their judgements on separate issues into one single vote for a (party) candidate that represents their preferred policy programme. The resulting policy programme or majority outcome can be said to be preferred *overall* by the majority of voters on the basis of their *aggregated* votes.[10]

The three majority voting paradoxes arise when the two methods lead to conflicting outcomes. In this section, we will discuss each of the three majority voting paradoxes and conclude with an answer to the question of why the paradoxes occur.

2.1. The Ostrogorski Paradox: the disadvantaged voter

The Ostrogorski paradox shows that the two methods of determining the majority vote on a full policy programme – parliamentary elections and the combination of referendum votes – may lead to conflicting outcomes. It shows that the *overall* majority outcome – the outcome based on the *aggregated* votes in parliamentary elections – can negate the *issue-wise* majority outcome – the outcome based on the *disaggregated* votes in referendums. In other words, while a political party may gain an overall majority in parliamentary elections, it might also be the case that none of its policies gains a majority when they are separately voted on in referendums. For example, if the policy issues were voted on in referendums in France, Emmanuel Macron's party *La Republique En Marche!* might end up executing the policies of the *Front National*, the nationalistic and increasingly Eurosceptic party of Marine Le Pen, who was Macron's rival in the presidential elections of 2017.

Table 1: The Ostrogorski Paradox

Voters	Issue 1	Issue 2	Issue 3	Parliamentary Vote
A (20%)	X	X	Y	X
B (20%)	X	Y	X	X
C (20%)	Y	X	X	X
D (20%)	Y	Y	Y	Y
E (20%)	Y	Y	Y	Y
Referendum Vote	Y	Y	Y	

Table 1 illustrates the Ostrogorski paradox. Suppose there are three policy issues and two political parties (X and Y). Also, suppose voters are divided into five groups (A, B, C, D and E), each comprising 20 percent of the electorate, and each voter votes for the party that they favour on most issues in parliamentary voting. Table 1 shows which political party is favoured by the groups of voters per issue. For example, voters in group A favour party X on Issue 1.[11]

Consider the parliamentary voting outcome. Since voters vote for the party that they favour on most issues in parliamentary elections, groups A, B and C would vote for party X and groups D and E would vote for party Y, because the first three groups favour party X on 2 out of the 3 issues and groups D and E favour party Y on all issues. Thus, party X gains 60 percent of the votes and wins the parliamentary election.

However, if the issues are considered separately, party Y will get 60 percent of the votes on each issue and will, hence, beat party X on all issues. For example, voters in groups C, D and E, comprising 60

---

percent of the electorate, favour party Y on issue 1. Similarly, on issues 2 and 3, party Y is favoured by 60 percent of the electorate. This means that the joint outcome of the referendums (party Y wins on all issues) negates the parliamentary voting outcome (party X wins in parliamentary voting).[12]

It should be noted that this contradiction between the overall majority and issue-wise majority outcome occurs solely because a different method of determining the majority opinion is used and not because a different group of people cast a vote or because people have changed their opinions.

This paradox shows that the method of determining the majority opinion may constitute a decisive factor in the outcome; whether a political party or policy programme is voted on in parliamentary elections or in referendums could determine which political party or policy programme is dominant. Consequently, the question arises which majority outcome reflects the majority opinion: the overall majority outcome in parliamentary elections or the issue-wise majority outcome in referendums? For example, does the French population favour the liberal and pro-European policies of Macron's *La République En Marche!* given that Macron won the presidential elections and his party won the parliamentary elections, or is the French population rather conservative and Eurosceptic if the policies of Le Pen's *Front National* are favoured issue-wise in referendums?

In the debate about the supplementary value of referendums, this paradox provides a counter-argument against the view that referendums enhance citizens' power over decision-making. Proponents of referendums often argue that the direct and unmediated vote provides the highest degree of legitimacy for decisions and produces more accurate expressions of the citizens' will than a parliamentary vote electing representatives who make the decisions for them (see Butler and Ranney 1978, pp. 24f.). However, the Ostrogorski paradox implies that referendums do not necessarily give citizens more power, but a different kind of power; referendums allow citizens to cast a disaggregated vote besides their aggregated vote in parliament and, hence, complicate the voter calculus. Referendums create a new avenue for interpreting citizens' opinions. As the issue-wise majority outcome of referendums might negate the overall majority outcome in parliamentary elections, some voters might be better off without referendums. Take, for example, the voters in group A, who are in a minority on the majority of issues but would belong to the majority in parliamentary elections.

2.2. The Anscombe Paradox: the disadvantaged governmental coalition

Closely related to the Ostrogorski paradox is the Anscombe paradox, which specifically reveals the tension between coalition formation and referendum voting.[13] It explains why holding several referendums might lead to frustration among the supporters of the dominant parliamentary coalition. 'In a nutshell, it says that a majority of voters [represented by the governmental coalition] could be in

---

[12] While the paradox is presented in terms of a choice between two political parties, the paradox also applies to multi-party systems. The paradox can be adjusted to fit a system with more political parties, e.g. by changing the vote for party X/Y into a dichotomous yes/no vote.

[13] Although the Ostrogorski and Anscombe paradoxes look similar, they are not equivalent. Nurmi and Saari (2010, p. 34) showed that every instance of the Anscombe paradox is an instance of the Ostrogorski paradox. But not every instance of the Ostrogorski paradox is an instance of the Anscombe paradox. Nurmi (1999, p. 73f.) gives an example of an Ostrogorski paradox that is not an instance of the Anscombe paradox. In Nurmi's example, the parliamentary majority outcome negates the issue-wise majority outcome, but only one voter is on the losing side on a majority of issues.

a minority (i.e. on the losing side) on a majority of issues involving dichotomous choices' (Nurmi 1997, p. 34).

Table 2: The Anscombe Paradox

	Voters	Issues 1	Issue 2	Issue 3
Coalition	A (20%)	Yes	Yes	No
	B (20%)	Yes	No	Yes
	C (20%)	No	Yes	Yes
Opposition	D (20%)	No	No	No
	E (20%)	No	No	No
Referendum	Outcome	No	No	No

Table 2 presents the Anscombe paradox.[14] Suppose there are again three policy issues and five groups of voters (A, B, C, D and E), each comprising 20 percent of the electorate. This time voters cast a yes/no vote. Table 2 shows the votes of each group per issue. Voters in group A vote 'yes' on issues 1 and 2 and 'no' on issue 3. The referendum outcome comprises the majority per issue. On all three issues the majority of voters (60 percent) vote 'no', which will be the referendum outcome.

Voters in groups A, B and C, a majority of voters (comprising 60 percent of the electorate), are on the losing side two out of the three times, while voters in groups D and E, a minority of voters (comprising 40 percent of the electorate), are always on the winning side.

Now, the point of the paradox is that even though a coalition supported by a majority of voters (groups A, B and C) would approve of all three issues, they might lose on all three issues in a referendum vote. To see this, suppose that the five groups of voters correspond to parties, each gaining 20% in parliamentary elections. In that case, no party has an overall majority in parliamentary elections and coalition formation is needed. Parties D and E are happy to enter into a coalition with each other as they agree on all policy issues. However, none of the other three parties wants to join them in a coalition as they disagree with parties D and E on the majority of issues. Therefore, the only possible majority coalition is a coalition with parties A, B and C. Parties A, B and C reach a coalition agreement in which each party renounces only 1/3 of their programme (i.e. one issue), resulting in a policy programme in which all policy proposals are accepted. Hence, this majority coalition would push through a policy programme that, on each policy issue separately, is against the will of the majority of voters.

This paradox addresses the concern of opponents of referendums that referendums might threaten the power and prestige of elected authorities (see e.g. Butler and Ranney 1978, p. 34). The paradox shows that not only may a single referendum outcome run counter to the carefully negotiated consensus of governmental coalitions but also that the combination of several referendum votes might lead to a policy programme that negates the policy programme carefully negotiated by the dominant parliamentary coalition. This might threaten the power and prestige of representatives. The significance of this paradox lies in the fact that this threat to a majority government arises only because a voting system is used that is fundamentally different from the voting system used in parliamentary elections, and not because different citizens with different opinions have cast their vote.

---

[14] Our presentation of the paradox follows the presentation by Saari (2008, p. 31).

2.3. Nurmi's Representation Paradox: conflict for representatives

Nurmi's representation paradox shows that representatives might face a norm conflict when the two methods of determining the majority opinion produce conflicting outcomes. With conflicting majority outcomes, representatives have to decide whether they want to represent either their supporters or the whole of people. In the former alternative they use the method determining the overall majority outcome – that is, they listen to the parliamentary voting outcome. In the latter alternative they follow the issue-wise majority opinion and, thus, respect the referendum results.

Table 3: The Representation Paradox

	Party A (2/3)				Party B (1/3)		Referendum Vote
	MP1	MP2	MP3	MP4	MP5	MP6	
Yes	5	5	5	5	11	11	**42**
No	6	6	6	6	0	0	24
Parliamentary Vote	**No**	**No**	**No**	**No**	Yes	Yes	

Table 3 presents Nurmi's representation paradox.[15] Suppose that there are two political parties (party A and party B) and that parliament consists of 6 members. Party A has 4 members in parliament, which is 2/3 of the parliamentary seats, and party B provides the other 2 members of parliament, which is 1/3 of the seats. Assume that this proportionally reflects the opinion of the electorate. That is, 2/3 of the electorate favours party A and 1/3 favours party B. Furthermore, suppose that the electorate consists of 66 voters. Given our proportionality assumption, this means that each group of 11 voters is represented by one member of parliament.

Suppose that a referendum is held in which the voters can choose either 'yes' or 'no'. Table 3 shows that 6 voters out of the groups of 11 voters that elected a member of party A vote 'no', the other 5 voters choose 'yes'. The voters in the groups for the members of party B unanimously choose 'yes'. This gives a majority of 42 'yes' votes out of the 66 votes in a referendum.

However, if in parliamentary voting the members vote in accordance with the majority opinion of their supporters, the opposite is the result. Assuming that they know the distribution of opinions of their supporters, the members of party A (members 1 to 4) will vote 'no', since the majority of supporters for each member prefer 'no' and, correspondingly, the majority of supporters for party A (24 of the 44 supporters) prefer 'no'. This results in 'no' as the outcome of parliamentary voting with a 2/3 majority.

The significance of this paradox is 'in the shadow it casts on the institution of consultative referendum' (Nurmi 1999, p.78). In consultative referendums, representatives are faced with the decision whether the parliamentary vote or the referendum vote is more decisive. The paradox shows that this decision may create a norm conflict: are they supposed to represent the whole people or just their own supporters? As Nurmi has explained: 'If the former alternative is the case, then the legislators have clear moral reasons to honor the referendum outcome. If, however, the MPs [members of parliament]

---

[15] We use a version of Nurmi's representation paradox that is slightly different from the versions presented by Nurmi himself. Nurmi presents the paradox with 200 members of parliament (1999, p. 77) and with 9 members of parliament (1997, pp. 48ff.; 1998, pp. 336ff.). We have presented the paradox with 6 members of parliament because this results in a table that is slightly easier to comprehend. Furthermore, note that this paradox could also have been formulated in terms of more than two political parties. Party A would then constitute the coalition parties while Party B would encompass the opposition parties.

represent their own supporters, then the referendum outcome is of no consequence to their actions' (Nurmi 1997, p.49). The paradox implies that non-binding referendums might enhance the power of representatives instead of citizens' power, as they provide representatives with an additional avenue for interpreting the majority opinion. However, this power goes hand in glove with a norm conflict.

2.4. Why do the paradoxes occur?

As the paradoxes produce surprising and maybe even counter-intuitive results, one might wonder: why do these paradoxes occur? To answer this question, it is useful to understand the similarities between the three voting paradoxes and Simpson's paradox. Simpson's paradox is a statistical paradox that was first addressed in a paper by Edward H. Simpson (1951, pp. 238ff.). The paradox shows that the conclusions drawn from a dataset may reverse when switching between interpreting the data as a whole (the *aggregated* dataset) and interpreting the data as a partitioned dataset (the *disaggregated* dataset). An actual occurrence of this paradox was observed by Cohen and Nagel (1964) in a comparison of the number of tuberculosis deaths in New York City and Richmond, Virginia, during the year 1910. Whereas the overall tuberculosis mortality rate was lower in New York than in Richmond, the opposite was observed when the data was divided in the racial categories of white and non-white people: New York had a higher tuberculosis mortality rate in each category than Richmond.

The three voting paradoxes also show this conflict between the conclusion drawn from the dataset as a whole and the dataset in its partitioned form. In the Ostrogorski and Anscombe paradoxes, the data of voter preferences is partitioned into issue-wise preferences. In Nurmi's representation paradox, the voter preferences are categorised according to the political party or the member of parliament that represents the voter.

Now, we can return to the question of why the paradoxes occur. There is quite a clear answer to this question: the paradoxes arise either because (1) the disaggregated dataset establishes connections in the dataset that the aggregated dataset cannot recognise and, hence, cannot respect, or because (2) the disaggregated dataset cannot recognise and, hence, cannot respect the connections among the subcategories that the aggregated dataset does recognise and respect. The first reason applies to Nurmi's representation paradox. In this paradox, the party affiliation of voters is recognised and respected in the parliamentary vote, but not in the referendum vote. The second reason applies to the Ostrogorski and Anscombe paradoxes: in issue-wise voting (referendums) the connection among separate issues cannot be recognised and, hence, cannot be respected, while this connection may very well play a role in the parliamentary vote.

We will continue our discussion by focussing on the Ostrogorski and Anscombe paradoxes, and the inability of issue-wise voting to recognise and respect connections among issues.

Table 4: Saari's Example

(a) The candidates

Engineering	Social Sciences	Sciences
David	Harry	Fred
Elaine	Joyce	Gloria

(b) Dean Votes

	Engineering	Social Sciences	Sciences
Dean 1	Elaine	Harry	Fred
Dean 2	David	Joyce	Fred
Dean 3	David	Harry	Gloria
Majority Outcome	David	Harry	Fred

The inability to respect relations among separate issues is illustrated by an example due to Saari (2008, pp.40ff.). Suppose a university panel should be assembled. Each of three schools put forward two candidates. To ensure an appropriate balance across disciplines, one candidate from each school will be selected. Table 4(a) shows the candidates and schools.

The deans of the schools vote for one candidate from each school's list and the majority winner from each list is elected. Table 4(b) shows the votes of the three deans. Each dean has voted for a panel composition of at least one man and one woman. Assume that each dean also prefers a panel composition of mixed gender. The issue-wise majority outcome, however, results in a panel of men only. As the issue-wise majority outcome does not respect the extra condition of mixed gender, each dean would be dissatisfied with this outcome.

By contrast, if the university panel was assembled on the basis of a parliamentary voting system, the deans would have voted for a full panel composition and would, hence, have only chosen a panel composition of mixed gender.

The upshot of this is that the issue-wise majority vote may produce a joint outcome that *nobody* really likes and that violates imposed conditions for the relations among the separate issues which are taken into account in the parliamentary vote.[16] In the Ostrogroski paradox, voters are asked to give a party preference for each party separately, while voters usually tend to base their party preference on their opinions about several issues. In the Anscombe paradox, the connection of belonging to the majority coalition is neither recognised nor respected in the issue-wise majority vote. 'Consequently, the outcomes are connected to the majority or minority party strictly by coincidence; the [issue-wise] majority vote has stripped away all membership connections' (Saari 2008, pp. 42f.).

The Ostrogorski and Anscombe paradoxes have a similar structure to the *discursive dilemma*, which has motivated research into judgement aggregation.[17] The discursive dilemma also arises 'when, in a

---

[16] By using preferential rankings it can be shown that the inability to respect connections among the separate issues results in an issue-wise majority outcome in the paradoxes that violates the mathematical relation of transitivity in preferences. Even though the individuals have transitive preferences, the issue-wise majority outcome might be cyclic. By contrast, the overall majority outcome will always be transitive as voters vote for the full programme and, hence, will incorporate their transitive preferences into their vote.

[17] For an introduction into judgement aggregation, see List and Puppe (2009) but also Endriss (2016). Judgement aggregation concerns the aggregation of belief information. It provides a relatively general framework, in which the social choice framework of preference aggregation can be embedded. There are different ways to embed the social choice framework into the judgement aggregation framework. For an embedding into a formula-based

group, each individual consistently makes a judgement, or expresses a preference, (in the form of yes or no) over specific propositions, and the collective outcome is in some respect inconsistent' (Pigozzi 2006, p. 119).[18] However, while this inconsistent collective outcome is a purely logical relationship among the propositions in the discursive dilemma, it is a compound majority decision that binds voters and issues in the Ostrogorski and Anscombe paradoxes (see Pigozzi 2010, but also Grandi 2014).

To conclude, Nurmi's representation paradox is troubling because the preferences of voters are connected to the party affiliation of the voters. Such a connection may only be reasonable if the referendum issue is also considered in the policy programmes of the political parties. The Ostrogorski and Anscombe paradoxes 'are troubling because we expect connections across pairs, as identified with voter preferences, to survive the decision process. Instead, the [issue-wise] majority vote completely severs all connections' (Saari 2008, p. 42), while these connections are taken into account in the overall majority vote. Because referendums delink the issues, referendums in representative democracies are not just an *additional* instrument, but a *different* instrument through which citizens can make their voice heard. As Nurmi (1997, p. 47) says, referendums 'are both an additional avenue for expressing one's opinion about specific issues and a complication in the voter calculus.'

3. Practical Implications

As we dusted off the three majority voting paradoxes, we have clarified the systemic differences between referendum voting and parliamentary elections, and enhanced our understanding of the consequences of implementing referendums in the representative system. Although the negative consequences suggest that caution should be exercised when combining parliamentary elections with referendums, they do not necessarily lead to the conclusion that the parliamentary system cannot and should not be supplemented by the direct voting system of referendums. Therefore, legislators and politicians might ask how they can use this knowledge of the systemic differences when deciding to implement referendums in a representative system. This section purports to address this question by formulating conditions that could prevent the paradoxes and, hence, the negative consequences of the paradoxes from arising. We will discuss four possible conditions.

First, we will argue that stipulating the legally binding nature of referendums could remove the representatives' norm conflict of Nurmi's representation paradox, but this is insufficient to deal with the Ostrogorski and Anscombe paradoxes.

Second, we will argue that the sequential use of referendums could resolve the norm conflict arising from Nurmi's representation paradox when the referendums concern bills that are approved of by a majority in parliament. However, sequential referendums cannot be a solution to the Ostrogorski and

---

framework of judgement aggregation, see Dietrich and List (2007, pp. 26ff.). For an embedding into a constraint-based framework of judgement aggregation, see Grandi and Endriss (2011).

[18] In particular, the discursive dilemma and the Ostrogorski and Anscombe paradoxes are essentially equivalent to the *paradox of voting*. The paradox of voting, also known as Condorcet's paradox, shows that a voter profile with three or more voters and consistent voter preferences can lead to a majority cycle, i.e., an inconsistent majority outcome. Nurmi and Saari (2010) proved that with a renaming of the alternatives the Ostrogorski paradox can be translated into the paradox of voting and *vice versa*. Saari (2008, ch. 2) showed that the Anscombe paradox and the discursive dilemma can be translated into the paradox of voting. List and Pettit (2002, pp. 89ff.) showed that the discursive dilemma is a generalisation of the paradox of voting.

Anscombe paradoxes or to Nurmi's representation paradox when the policy proposal is rejected by a majority in parliament but is accepted in a referendum.

Third, as stipulating that referendums be legally binding or using referendums sequentially cannot resolve the Ostrogorski and Anscombe paradoxes, we will examine whether a quorum requirement could. We will discuss Wagner's (1983) result that an approval quorum of 75% for referendums could prevent the Anscombe paradox from arising, but show with Deb and Kelsey's (1987) result that this is insufficient to deal with the Ostrogorski paradox because of necessary additional requirements.

Fourth, we discuss our last possible condition, which aims at avoiding connections between referendum issues and the policy programme elected in parliamentary elections. We argue that the Ostrogorski and Anscombe paradoxes can be prevented from arising if referendums are only implemented when the referendum issues do not have a connection to the policy programme elected in parliamentary elections. Suggestions will be made on how this condition can be satisfied.

Before going into the practical implications, we would like to deflect the charge that the occurrence of these paradoxes is not frequent at all and that it might, therefore, not be worth basing conditions for the application of referendums on the results of these paradoxes. The question of the frequency of the paradoxes can only be answered if more information about voter preferences is available. But, irrespective of the frequency of their occurrence, the paradoxes show robust systemic differences and negative consequences that can (partially) explain, *inter alia*, frustration of citizens and representatives following the Brexit,[19] Ukraine[20] and Hungarian migrant quotas[21] referendums.

3.1. Binding Referendums

Nurmi (1997, pp.25ff., p.51) argues that the problems arising from combining referendums with parliamentary elections 'can be overcome through the requirement that the referenda be binding, i.e. not consultative'. Specifically, he concludes that requiring a binding nature is a way of handling his representation paradox. But would stipulating that referendums be legally binding also prevent citizens' frustration resulting from an Ostrogorski paradox and the problems of majority coalitions in parliament following from an Anscombe paradox?

The problem with non-binding referendums is that they allow the two methods of determining the majority outcome to coexist and leave it to the government to decide, *ex post*, the authoritativeness of either method. Non-binding referendums give the government, *de jure*, two options for interpreting the referendum results: the government can either accept the referendum outcome and, thus, consider the issue-wise majority to be the most authoritative or reject the outcome and, thus, regard the overall majority outcome as the most authoritative.

The Ukraine referendum shows that non-binding referendums even open up a third path, the legality of which is – at least in the Netherlands – completely excluded from any judicial scrutiny. The Dutch government chose neither to accept nor to reject the outcome of the Ukraine referendum directly, as it did not immediately submit a legislative proposal to accept or withdraw the Approval Act on the EU-

---

[19] The referendum of the United Kingdom on European Union membership, June 2016.
[20] The referendum of the Netherlands on the approval of the Association Agreement between the European Union and Ukraine, April 2016.
[21] The Hungarian referendum on the migrant relocations plans of the European Union, October 2016.

Ukraine Association Agreement.[22] Instead, Prime Minister Rutte invented his so-called *desire path* by using the referendum outcome as a basis for further negotiations on the Association Agreement in the European Council. The legality of this path was contested at the The Hague district court by *Stichting Forum voor Democratie* (case *Stichting Forum voor Democratie* v. *De Staat der Nederlanden,* 12th April, 2017). The judge held, however, that he was not allowed to decide on the lawfulness of the government's actions, because he ought not to interfere with this process of political decision-making.[23]

By leaving it to themselves to decide which method of analysing the majority opinion is the most authoritative, governments face the norm conflict of Nurmi's representation paradox in non-binding referendums. For example, did the British government in the Brexit referendum represent their supporters and were their actions, therefore, legitimised by the parliamentary elections? Or, as Theresa May clearly chose to see it, did the government have to represent the whole population – assuming that the referendum majority was representative of the population's opinion – and listen to the referendum outcome? With the 2019 local elections in the UK, this picture becomes even more complicated. As Brexit was among the main themes of the election campaign, it is not clear whether the Conservatives' defeat should be interpreted as a vote against a 'hard' Brexit or as a lack of support for their full policy programme.

The coexistence of the two methods and the political discretion on the authoritativeness of either method in non-binding referendums causes uncertainty among citizens, because it is only after the referendum that they find out whether their vote was authoritative. Furthermore, the political choice of either method is likely to lead to frustration among citizens. On the one hand, large groups of voters might feel frustrated when the majority opinion is established by analysing the issue-wise votes in referendums, because then their vote in parliamentary elections is not reflected in political decisions. On the other hand, large majorities of voters might feel frustrated when the decisions are based on the overall majority outcome in the parliamentary elections instead of the issue-wise majority opinion in the referendums, because then their referendum vote appears to be useless.

In referendums with a legally binding nature it is determined, *de jure*, that the referendum outcome is more authoritative than the outcome of the parliamentary elections and the acceptability of inventive desire paths would be under legal scrutiny. This takes away the norm conflict of Nurmi's representation paradox, because if referendums have a legally binding nature then the law prescribes that the

---

[22] Article 11 of the (now abolished) Dutch Consultative Referendum Law pointed out that there are two options when the majority rejects its legislation in a non-binding referendum: (i) accepting the referendum outcome and submitting a legislative proposal to withdraw the legislation, or (ii) rejecting the referendum outcome and presenting a legislative proposal to enact the legislation. The Article did not define by whom the legislative proposal should be submitted. On the basis of Art. 82 of the Dutch Constitution, both the government and the Second Chamber can submit a legislative proposal. Legislative proposals of the Second Chamber are also called initiative proposals (*initiatiefvoorstellen*). In the case of a rejecting referendum outcome it is most natural that a governmental legislative proposal rather than an initiative proposal is submitted. This is also reflected in the explanatory memorandum to the law, which states: 'artikel 11 verplicht de regering' [Article 11 obliges the government] (*Parliamentary Papers* 2005/06, p. 19).

[23] Article 81 of the Dutch Constitution allots responsibility for enacting Acts of Parliament solely to the Government and the States General. In the *Tegelen* judgement (Supreme Court of the Netherlands, 19th November, 1999, para. 3.4) it was ruled that this also prevents judges from deciding on the procedures for enacting laws. Judging on the procedures would allow a judge to influence the legislative procedure by ruling whether the procedures have been followed lawfully and, if not, what consequences should result from that.

referendum outcome should be honoured and, thus, that the whole people should be represented. The legally binding nature prevents representatives from adapting their ideology of representation to the situation and, hence, takes away any uncertainty among citizens concerning the voter calculus.

However, determining that the disaggregated vote is more authoritative goes hand in glove with reducing the strength of the parliamentary vote. With legally binding referendums, majority government coalitions have to stick to the referendum results that frustrate their carefully negotiated policy programme, as in the Anscombe paradox.

Thus, stipulating that referendums be legally binding is sufficient to take away the government's norm conflict and citizens' uncertainty concerning the authoritativeness of their vote, but it also allows no leeway when the referendums disrupt the representative system. The legally binding nature magnifies the problems a majority coalition is facing when it is in a minority on a majority of issues, as in the Anscombe paradox. Therefore, it cannot be a complete solution to the problems arising when combining referendums with parliamentary elections.

Besides, even if stipulating the legally binding nature of referendums would be a solution to the problems arising from the paradoxes, it may be a politically unfeasible requirement, especially in countries with constitution that do not allow for legally binding referendums.[24] For example, Article 81 of the Dutch Constitution confers legislative power to the Government and the States General jointly and, thereby, excludes citizens from enacting legally binding acts. As a consequence, Article 81 of the Dutch Constitution has to be amended to introduce legally binding referendums. However, passing the constitutional amendment procedure in the Netherlands is notoriously difficult. A constitutional amendment has to be passed twice in both the Upper and Lower House of Parliament and has to be ratified twice by the government. This is referred to as the 'two readings'.[25] Three times an amendment of Article 81 of the Dutch Constitution has been proposed in order to allow for binding referendums, and each time the bill failed to acquire the support of a qualified majority of two thirds in either the Upper or Lower House of Parliament in the second reading.[26] The question is when, if ever, Dutch politicians will be willing to consider a new constitutional proposal for binding referendums.

---

[24] In Europe, these countries include Belgium, Finland, the Netherlands and the United Kingdom.

[25] The first reading takes the form of the procedure for a normal Act of Parliament: the constitutional amendment has to be supported by a simple majority in each House of Parliament. After the publication of the bill for the constitutional amendment, the Lower House has to be dissolved. Usually, the dissolution of the Lower House is timed such that it coincides with the periodic general elections of the Lower House and the formation of a new government. In the second reading, each House of Parliament has to support the constitutional amendment with a qualified majority of two-thirds for the amendment to pass. The process of amending the Dutch Constitution is determined by Article 137 of the Constitution. For more information, see Besselink (2007).

[26] The first bill for amending Article 81 of the Dutch constitution was rejected in second reading in 1998, leading to the resignation of Cabinet Kok-II. The second bill for amending Article 81 of the Dutch Constitution was submitted to the first reading in 2000 and failed to acquire the required support in the second reading in 2004. The third bill for a constitutional amendment was rejected in second reading in November 2017 under Cabinet Rutte-III. This government also repealed the Consultative Referendum Law in response to the turmoil that arose after the first application of the law in the referendum on the EU Association Agreement with Ukraine. For more information, see Breunese (2013, pp. 346ff.) and Bovend'Eert (2018).

3.2. Sequential Referendums

As an alternative to stipulating that referendums be legally binding, an instance of Nurmi's representation paradox could be avoided by holding several referendums on the same issue. We call this: the sequential use of referendums.

Two instances of Nurmi's representation paradox should be distinguished. In the first instance, a policy proposal is accepted by a majority in parliament but is rejected in a referendum.[27] The members of parliament face the consequences of Nurmi's representation paradox: they have to choose between representing their own supporters (i.e., ignoring the referendum outcome) and representing the whole people (i.e., honouring the referendum outcome). The second instance of Nurmi's representation paradox is the mirror image of the first instance in terms of the referendum result and the parliamentary voting outcome. This instance of Nurmi's representation paradox concerns a policy proposal that is rejected by a majority in parliament but is accepted in a referendum.[28] Again, members of parliament face a norm conflict, as the referendum result and the outcome of the parliamentary vote suggest different actions.

Sequential referendums can be a solution to the first of these two instances of Nurmi's representation paradox. In the first instance, members of parliament face a choice between honouring the parliamentary voting outcome that approves of the policy proposal and honouring the referendum outcome that rejects the policy proposal. Now, with a practice of sequential referendums, choice can be avoided as a (revised) policy proposal can be submitted to a new referendum until it is also accepted in a referendum vote. This is common practice in Switzerland, where the Parliament has the option to submit a bill, in a revised version, to a new referendum when the original bill is rejected in a referendum. The revision of the bill should be based on the results of a consultation of the citizens about their reasons for rejecting the bill. There is no limit to the number of times that the Swiss Parliament can revise and submit the revised bill to a new referendum. That is, the Parliament can continue this process of revising and resubmitting until the bill is accepted. However, the sequential use of referendums is not likely to be exhausted, as resubmitting costs time and money.

Hence, the practice of sequential referendums on parliamentary bills, that have been approved of by parliament, would be an alternative to stipulating that referendums be legally binding in order to prevent the norm conflict of Nurmi's representation paradox from arising. For example, submitting the re-negotiated EU-Ukraine Association Agreement to a referendum could have contributed to the legitimacy of Rutte's inventive desire path, provided that the second referendum on the agreement had resulted in the approval of the referendum voters. It should be noted that referendums have a *materially* binding nature with such a practice of sequential referendums, as government is expected to honour the referendum result. When a policy proposal is rejected in a referendum, government is expected to refrain from implementing the policy proposal until a revised version is accepted in a referendum.

By contrast, the second instance of Nurmi's representation paradox cannot be avoided by submitting a revised proposal to a new referendum as, in this instance, the policy proposal is already approved of

---

[27] Note that we can easily construct this instance of the representation paradox from Table 3 by changing the 'yes' votes into 'no' votes and *vice versa*.

[28] This instance is exemplified by Nurmi's representation paradox as presented in Table 3.

in the referendum.[29] The Swiss 2009 initiative to ban minarets[30] and the Swiss 2014 anti-mass migration initiative[31] are examples of policy proposals that were not supported by a majority in parliament but accepted in a referendum.

In theory, the members of parliament could avoid the paradox by revising the policy proposal until the majority of members in parliament also approve of the proposal. Such a revised proposal is, however, likely to face legitimacy issues when it is a substantial weakening of the initial proposal, as the question may arise whether the revised proposal can still live up to the wishes that the people expressed in the first referendum.

Alternatively, it may in exceptional circumstances be legitimate to overturn the referendum result and call for a second referendum. The Swiss Supreme Court has declared invalid several results of cantonal and local referendums over the past 100 years and since 2007, the Court has the right to rule on the validity of national referendums. In April 2019, the Swiss Supreme Court used this right to declare the outcome of a national referendum invalid for the first time in Switzerland's recent history. The referendum rejected by 50.8% to 49.2% the government's plans to reduce the tax burden for married couples.[32] In its landmark ruling, the Swiss Supreme Court overturned the referendum result on the grounds that the information given to the voters was intransparent and incorrect. In information provided to the electorate prior to the referendum, the federal government had said that 80,000 married couples were negatively affected by the tax burden. But following the referendum vote, the government conceded that it had seriously underestimated the numbers and said that, in fact, 454,000 married couples were affected by the tax burden. Although the Swiss Supreme Court's verdict has led to controversy, it may resonate in the United Kingdom, where Remain campaigners have consistently argued that voters were not adequately informed.[33]

As revising a policy proposal that is accepted in a referendum or directly overturning a referendum result is politically feasible in only exceptional circumstances, stipulating the binding nature of referendums on policy proposals that are rejected by parliament is a more general solution to prevent Nurmi's representation paradox from arising. In the world's most prominent referendum democracy, this recommendation is already followed. Consequently, the 2009 ban on minarets and the 2014 anti-mass migration initiative have been implemented, albeit in a lighter version.

Whereas the sequential use of referendums can prevent the instance of Nurmi's representation paradox from arising, in which the policy proposals is supported by a majority in parliament, such a

---

[29] Note that there is an anti-symmetry between the acceptance and the rejection of a policy proposal in a vote. The legitimacy of public decisions requires parliament to search for a policy proposal that is accepted in both the parliamentary vote and the referendum vote. Changing a policy proposal in order to have it rejected in a referendum vote is an unconstructive approach to law-making.

[30] The minaret initiative was launched by politicians form the Swiss People's Party and the Federal Democratic Union. It sought a constitutional ban on the building of new minarets on mosques.

[31] The anti-mass migration initiative was launched by the national conservative Swiss People's Party. It aimed at limiting immigration through quotas.

[32] As the income of married couples is taxed jointly and married couples get tax-free allowance over the combined income, they pay more tax than unmarried cohabiting couples who are taxed separately and therefore each have a tax-free allowance. The referendum concerned the government's plans aimed at ending this extra charge on married couples compared to unmarried cohabiting couples by allowing married couples to pay the tax that is calculated over their incomes separately.

[33] For newspaper articles about the Swiss Supreme Court's historic ruling, see Alder (10th April, 2019), Geiser (10th April, 2019), Kleck (11th April, 2019) and North (23rd April, 2019).

referendum practice cannot prevent the Ostrogorski and Anscombe paradoxes from arising, as these paradoxes may arise with respect to each referendum in the sequence. Since stipulating that referendums be legally binding also cannot prevent the Ostrogorski and Anscombe paradoxes from arising, we will discuss a potential solution to the latter two paradoxes in the next section.

3.3. Quorum requirement

One way of dealing with the Ostrogorski and Anscombe paradoxes might be to introduce an approval quorum in referendums. An approval quorum singles out the *status quo*, i.e. no change in legislation or states of affairs, as the favourite by requiring a qualified majority of votes for change. It requires that the majority in the outcome of the referendum consists of a certain percentage of the electorate. Another type of quorum requirement is a participation quorum. This quorum does not impose a percentage requirement on the majority outcome but on the number of participants in the referendum: it requires that a certain percentage of the electorate has participated in the referendum.[34]

The Anscombe paradox can be avoided by imposing an approval quorum requirement of 75% in referendums. Wagner (1983) has shown 'that if instead of the simple majority rule one would require a $3/4$ majority for a proposal to be adopted, then it cannot be the case that a majority of voters is on the losing side on a majority of adopted proposals' (Nurmi 1999, p.83). That is, when a policy proposal is only adopted if a $3/4$ majority votes 'yes' in the referendum on the proposal, then there is no majority coalition that disagrees with the majority of adopted proposals.

The argument contains an important nuance: Wagner is only concerned with *adopted* proposals. Consider the case in which 63% of the voters are in favour of a proposal. Under the three-fourths rule the proposal would not be adopted (63% is less than 75%) but clearly a majority of 63% of the voters disagrees with the result. Wagner is not concerned with eliminating this 'majority loss' for rejected proposals. This makes sense in legislative settings in which there is a *status quo* that is maintained when there is no qualified majority for change.

In the Anscombe paradox as presented in Table 2, this subset of adopted proposals is empty as the referendum outcome on all three issues is 'no'. Thus, the Anscombe paradox as presented in Table 2 is not a good example to illustrate that Wagner's three-fourths rule works. However, Table 2 is only one example of an Anscombe paradox. The paradox can be constructed in many more ways: with a different number of voters, a different number of issues and different voter preferences. The strength of Wagner's rule is that it works for any number *N* of voters that cast a yes/no vote on a set of *K* proposals and, hence, for any possible construction of an Anscombe paradox (see Wagner 1983, p. 305).

The Ostrogorski paradox can also be avoided by requiring an approval quorum of 75% in referendums. However, this only works if, in addition, there is a majority threshold in parliamentary voting.

Let us denote the majority threshold on the parliamentary level by *k*. If *k*=$3/4$, each voter has to agree with at least 3 out of the 4 issues in order to become a voter for the political party with that policy

---

[34] In fact, an approval quorum presupposes a participation quorum, for if the majority should encompass x% of the electorate then also *at least* x% of the electorate must have participated in the referendum - see e.g. Bloks (2018, p. 87).

programme. Let us denote the majority threshold on a referendum level by g. If $g=3/4$, then 75% of the voters have to vote 'yes' in order for the policy proposal in the referendum to be accepted.

Deb and Kelsey (1987) have established a relationship between a majority threshold in parliamentary voting (k) and a majority threshold in referendum voting (g), which is: $g < 1/(4k-2)$. If g is strictly smaller than $1/(4k-2)$, the Ostrogorski paradox can occur. From this relation between g and k it follows that when k is higher, a lower g is sufficient to avoid the Ostrogorksi paradox. In other words, the higher the majority threshold in parliamentary voting, the lower the threshold in referendum voting has to be in order to avoid the Ostrogorski paradox.

We will look at a few examples. If $k=3/4$ the required value for g is 1. This means that voters will only vote for a political party in parliamentary elections when they agree with at least 75% of the issues, and that the quorum requirement in the referendums should be 100%. For values of k higher than $3/4$, the required value of g is less. If $k=7/8$ it is $g=2/3$, and if $k=5/6$ it is $g=3/4$, the latter meaning that the quorum requirement in referendums should be 75%. For values of k less than $3/4$, the Ostrogorksi paradox is always possible, since $k<3/4$ requires $g>1$, but a majority support of more than 100% is impossible.

Thus, the Ostrogorski paradox can be avoided by imposing an approval quorum of 75% in referendums, but then the support for the governing party has to be very cohesive as voters have to agree with more than 83% (5/6) of the issues when supporting the party. Another possibility is, for example, the requirement that voters agree with 75% of the issues when they vote for a party in parliamentary elections and that a policy is only accepted in a referendum when 100% of the voters agree.

Although an approval quorum of 75% provides a mathematical solution to the Anscombe paradox and an approval quorum in referendums in combination with a cohesion requirement in parliamentary voting provides a mathematical solution to the Ostrogorski paradox, the question is whether these solutions are practically feasible and beneficial.

Firstly, an approval quorum can have other negative consequences, which means that these solutions replace one problem with another. The mathematician and social choice theorist May proved that every rule, which is not based on a simple majority decision, will lead to problems: it 'will either fail to give a definite result in some situation, favor one individual over another, favor one alternative over the other, or fail to respond positively to individual preferences' (May 1952, p.683).[35] An approval quorum is a deviation of the simple majority rule that creates a bias in favour of the *status quo*.

The consequences of an approval quorum are less obvious than the consequences of a participation quorum. It is well-known that a participation quorum can lead to voter abstention as some *status quo* voters choose to abstain from voting because their vote would help the voters for change as they help meet the participation quorum requirement. An example is the Ukraine referendum in which those in favour of the EU-Ukraine Association Agreement did not know whether it was better to stay at home or vote. In the end, the participation quorum was met and the abstention of *status quo* voters had contributed to a higher percentage of voters against the agreement. While an approval quorum does not lead to this type of strategic voter abstention, it can be shown that it also affects voter participation

---

[35] The formalisation of this is also known as May's Theorem.

in the referendum. An approval quorum leads to passive voter abstention as *status quo* voters might choose to abstain from voting because their vote is no longer pivotal.[36]

Secondly, the requirement of a 75% approval quorum will likely be politically unfeasible in any European country. A 30% approval quorum has already been called a 'Russian requirement' by parliamentarians in the Netherlands, because it demands a high majority for change (see Parliamentary Papers 1999/2000, p.12). The reason is that participation in referendums is usually not higher than 50%, which means that 60% of the voters must have voted for change with a 30% approval quorum and that a 75% approval quorum can never be met.[37]

Lastly, the requirement of cohesiveness in parliamentary elections for preventing the Ostrogorski paradox is unrealistic. It cannot be imposed on voters that they only cast a vote for a certain political party if they agree with a minimal number (75% or 83%) of the issues. Moreover, a 100% approval quorum for a referendum when there is a cohesiveness of 75% would make winning a referendum practically impossible and, therefore, cause too much resistance among proponents of referendums.

To conclude, an approval quorum of $3/4$ in referendums would prevent the Anscombe paradox from occurring and in combination with a requirement on the cohesiveness in parliamentary voting it would prevent the Ostrogorski paradox from occurring. These solutions are, however, either practically unfeasible or not preferable given other negative consequences.

3.4. Avoiding connections

Since a quorum requirement cannot offer a politically feasible solution to prevent the Ostrogorski and Anscombe paradoxes from arising, we will discuss in this subsection another condition that provides a potential solution to the two paradoxes. This condition will be called 'avoiding connections' and we will suggest referendum issues that could satisfy this condition.

Since the Ostrogorski and Anscombe paradoxes occur because the connection of the separate issues to a full policy programme or the connection of the outcome to belonging to the majority party is lost, this teaches us that issue-wise voting is only appropriate if these connections do not matter. Only if issues are separate from each other and are not related to party politics does the issue-by-issue alternative provide a good alternative. If relations over the issues are of value, referendums should not be introduced into politics. This mistake was made in the Brexit referendum and the Ukraine referendum where the government's position and the possibility of executing its policies hung on the referendum outcome. David Cameron had to end his career as Prime Minister because of the referendum outcome and the Dutch government was stuck with a referendum result that directly contradicted its policies, not to mention the fact that it flew in the face of the government's intentions expressed in the international community.

We will suggest some referendum issues that could satisfy this condition of avoiding connections.[38]

---

[36] See Bloks (2018, pp. 87ff.) and Aguiar-Conraria and Magalhães (2010a, p. 544; 2010b, p. 69) for more information about the consequences of participation and approval quora.

[37] Recall that an approval quorum presupposes a participation quorum, for if the majority must consist of 75% of the electorate, then *at least* 75% of the electorate must participate in the referendum (see Bloks 2018, p. 88, but also *Parliamentary Papers* 1999/00, p. 12).

[38] It should be noted that this only concerns issues which are not yet excluded by law. See Borman (2017, p. 173) for issues that can be excluded by law and the reasons for their exclusion in the Netherlands.

An important indication could be whether the issue is included in the governmental coalition agreement. If the issue forms part of the coalition agreement, it was probably dominant in the election campaigns and forms the basis of the government's authority gained in the parliamentary elections. Hence, such issues are connected to party politics.

Not necessarily all issues of the coalition agreement have to be excluded from referendums, since it is also possible that the governmental coalition states in the agreement which issues are suitable for a referendum and which issues are not. Thereby, it can exclude in advance the issues which are too closely connected to the authority they derive from the parliamentary elections and they can allow referendums on some less threatening issues.

The governmental coalition agreement is especially a good indication when the government forms a majority coalition. If the government forms a minority coalition, it faces legitimacy problems and the agreement might not have enough authority to exclude referendums on issues in the agreement.

The consequence of excluding the issues of the governmental coalition agreement from referendums is that citizens can no longer initiate a referendum on issues which were a hot topic during the parliamentary elections, for they have already had an opportunity to express their opinion on these topics. Indeed, the corrective referendum is ruled out on these issues.

When the issues in the coalition agreement are excluded, there might still be other issues that are suitable for a referendum. An example is a new important issue that arises in a crisis situation after the parliamentary elections. Suppose the euro falls and the government suddenly has to decide whether or not it should start a new monetary union with a select club of European countries. It might be wise for the government to consult the citizens in a referendum, or citizens might initiate a referendum on this topic.

Another example could be smaller issues that are considered important but are not a hot topic, such as whether we want to have elected mayors. These are usually not part of the coalition agreement or the government could state the acceptability of referendums on these issues in its agreement. With respect to such issues, however, the question could be raised whether they are relevant enough to be subjected to a referendum or whether enough attention has been devoted to the topic for citizens to be able to form an informed opinion.

The last example includes issues on which parliament decides to consult the citizens. If a majority coalition is formed on several issues, but no consensus has been reached on one specific issue, a referendum on this issue might be a good solution. The specific issue is then detached from the majority coalition and the party programmes, even though it might have been an important topic in the parliamentary elections. An example of a referendum held to resolve internal divisions within the governing party or coalition of parties is the United Kingdom European Communities referendum in 1975. 'When the British Labour Party came to power in 1974 shortly after British accession to the Community, it was split on the issue of membership of the European Economic Community and decided to call a referendum in 1975 to resolve the debate within the party' (Hobolt 2006, p. 157).

To conclude, if connections are to be avoided, the issues which are suitable for a referendum are severely limited and it is in the hands of the government after the parliamentary elections which issues are allowed in referendums. This restricts the corrective power of citizens in referendums, but it is the price that we, as citizens, have to pay for the advantages of our representative system.

4. Concluding Remarks

In the current debate about the supplementary value of referendums, believers and non-believers of referendums are at cross purposes; they weigh the pros and cons of referendums differently and emphasise different consequences of combining referendums with parliamentary elections. This paper aims to bring the opposite sides of the debate together by focussing on the structural features of referendums and parliamentary elections. As the systemic contrarieties between referendum voting and parliamentary elections and the consequences of these differences are laid bare, it becomes clear that there is truth in the arguments on both sides of the debate. Specifically, a closer look at the contingent features of the referendum system and the system of parliamentary elections elucidates that referendums neither always enhance the power of citizens over decision-making nor always threaten the power and prestige of representatives. Hence, the arguments on both sides of the debate should be nuanced, with common ground being found in a discussion whether, and if so, when the structural features of each voting system are desirable.

In response to these conclusions, it might be objected that the paradoxes make too stark assumptions about the way representatives act and, therefore, cannot accurately reflect the real implications for the power of citizens and representatives over decision-making. This objection relates to the "mandate-independence controversy" of representation, so illuminatingly coined by Pitkin (1967). Our interpretation of the mathematical paradoxes seems to assume that representatives are bound by mandates and (have to) do exactly what their constituents want. But this is only one pillar of the concept of representation. The concept of representation also includes an idea of independence, where representatives should or must be free to act as seems best to them in pursuit of their constituents' welfare. When representatives follow their own judgement and not necessarily their constituents' vote, the demands of the voters might not be fully translated into parliamentary decisions, the policy programmes might in reality be more in line with the referendum results than the paradoxes portray and, more importantly, referendums might provide the only unmediated power of citizens over decision-making.

However, independent of empirical assumptions about the way representatives act, a fundamental result follows from the paradoxes concerning an incoherence of the citizens' will on policy programmes. The mathematical paradoxes show that, contrary to what many people believe, the direct and unmediated vote in referendums does not necessarily provide a more accurate expression of the citizens' will than does the outcome of parliamentary elections. Referendums only provide a different expression that may result in a different – even contradicting – majority outcome.[39] In fact, this different and direct expression of the citizens' will may lead to a majority outcome that nobody really likes when several referendums are held on connected issues. Hence, the claim that the referendum vote, although unmediated, does not necessarily give more but a different power to citizens follows from the mathematical paradoxes independently of a background theory of representation.

---

[39] As noted in footnote 9, we assume that the citizens' vote is based on voters' attitudes towards the specific referendum topic and is not a 'second-order' election vote that is a means of signalling satisfaction or dissatisfaction with the government. If the latter were the case, the referendum vote would not constitute a different power, but a second way of casting a parliamentary vote.

This insight has normative implications: when taking parliamentary elections as the *status quo*, the systemic differences between parliamentary elections and referendums provide a strong argument for being cautious about supplementing parliamentary elections with referendums. The paradoxical situations to which the combination of the two different voting methods can give rise, blur the expression of the citizens' will. Therefore, it is questionable whether citizens should be given the opportunity to cast a direct vote in a representative system.

Although the systemic contrarieties highlight that supplementing parliamentary elections with referendums may lead to problems, it cannot be concluded that referendums can never and should never be introduced in a representative system. Several conditions have been outlined in this paper that make it possible to overcome the conflicts between the two voting systems:

(1) In order to prevent Nurmi's representation paradox from arising, referendums should either have a legally binding nature or be used sequentially in case of referendums on bills that have been approved of in parliament, giving them a materially binding nature. The former requirement could be politically unfeasible in countries whose constitutions do not allow for legally binding referendums. The latter requirement is more likely to be politically feasible but cannot avoid the norm conflict of Nurmi's representation paradox from arising when a referendum is held on a policy proposal that is rejected by a majority in parliament.

(2) The referendum issue should not be connected to the policy programme elected in parliamentary voting. An indication could be whether the issue is included in the governmental coalition agreement, as these issues constitute the main themes in the parliamentary elections from which the government derives its authority.

(3) The referendum can have an approval quorum requirement of 75% to deal with the Anscombe paradox, but this is not necessary if condition (2) is met. Also, given practical limitations and the negative side-effects of quorum requirements, a referendum with an approval quorum is not preferable to a referendum based on simple majority rule.

As these conditions severely restrict the application scope of referendums in the representative system, this might give rise to the criticism that such a restrictive application of referendums will not be enough to respond to the current challenges to political legitimacy in European countries. Be that as it may, the focus on structural features suggests that improving the representative system from within or creating new forms of democracy that combine different features in one system[40] is more likely to enhance political legitimacy than does fogging the political debate by combining the uncombinables.

---

[40] An example of a new form of democracy that combines different features of direct and indirect democracy in one system is *delegative* or *liquid democracy*. This is a form of democracy where voters can vest their voting power in delegates instead of voting directly themselves. See, e.g., the *Liquid Democracy Journal,* published by Interaktive Demokratie e. V., Berlin, Germany. Also, see Brill (2018) for a computational social choice introduction into liquid democracy, and see Blum and Zuber (2016) for a philosophical paper on liquid democracy.

References

Literature

Aguiar-Conraria, L. and Magalhães, P.C. (2010a). 'How Quorum Rules Distort Referendum Outcomes: Evidence from a Pivotal Voter Model', *European Journal of Political Economy* 26 (4): 541-557.

Aguiar-Conraria, L. and Magalhães, P.C. (2010b). 'Referendum Design, Quorum Rules and Turnout', *Public Choice* 144 (1): 63-81.

Alder, K. (2019). 'Bundesgericht Annulliert Abstimmung zur Heiratsstrafe', *Neue Zürcher Zeitung*, 10th April, URL = https://www.nzz.ch/schweiz/heiratsstrafe-ld.1473970 (accessed on 23rd May, 2019).

Anscombe, G.E.M. (1976). 'On Frustration of the Majority by Fulfilment of the Majority's Will', *Analysis* 36 (4): 161-168.

Arrow, K.J., Sen, A. and Suzumura, K. (eds.) (2002) *Handbook of Social Choice and Welfare* (Amsterdam: Elsevier), volume 1.

Arrow, K.J., Sen, A. and Suzumura, K. (eds.) (2011) *Handbook of Social Choice and Welfare* (Amsterdam: Elsevier), volume 2.

Besselink, L.F.M. (2007) 'Constitutional Referenda in the Netherlands: A Debate in the Margin', *Electronic Journal of Comparative Law* 11 (1) : 1-23..

Bloks, S.A. (2018). 'De Wet Raadgevend Referendum Afschaffen of Verbeteren?' ['Abolishing or Improving the Consultative Referendum Law?'], *Nederlands Juristenblad* 2 : 85-91.

Blum, C. and Zuber, C. I. (2016). 'Liquid democracy: Potentials, problems, and perspectives', *Journal of Political Philosophy* 24 (2) : 162-182.

Borman, T. (2017). 'Niet-Referendabele Onderwerpen in de Wet Raadgevend Referendum' ['Topics that Cannot be Submitted to a Referendum in the Consultative Referendum Law'], *Rechtsgeleerd Magazijn Themis* 4: 173-182.

Bovend'Eert, P. (2018). 'How the Dutch Cannot Make Up Their Minds on Accepting Referendums', *IACL-AIDC Blog*, URL = https://blog-iacl-aidc.org/blog/2018/7/16/how-the-dutch-cannot-make-up-their-minds-on-accepting-referendums (accessed on 23rd May, 2019).

Brams, S. J., Kilgour, D. M. and Zwicker, W. S. (1998). 'The Paradox of Multiple Elections', *Social Choice and Welfare* 15 (2) : 211-236.

Breunese, H.M.B. (2013). 'Het Referendum: Nieuwe Ronde, Nieuwe Kansen' ['The Referendum: New Round, New Opportunities'], *Tijdschrift voor Constitutioneel Recht* 4 : 346-353.

Brill, M. (2018). 'Interactive democracy', *Proceedings of the 17th International Conference on Autonomous Agents and MultiAgent Systems (AAMAS)* : 1183-1187.

Butler, D. and Ranney, A. (1978). *Referendums. A Comparative Study of Practice and Theory* (Washington, D.C.: American Enterprise Institute for Public Policy Research).

Cohen, M.R. and Nagel, E. (1964). *An Introduction to Logic and Scientific Method*. (New York: Harcourt, Brace and World, Inc.).

Deb, R. and Kelsey, D. (1987). 'On Constructing a Generalized Ostrogorski Paradox: Necessary and Sufficient Conditions', *Mathematical Social Sciences* 14 (2): 161-174.

Dietrich, F. and List, C. (2007). 'Arrow's Theorem in Judgement Aggregation', *Social Choice and Welfare* 29 (1): 19-33.

Endriss, U. (2016). 'Judgement Aggregation', in F. Brandt, V. Conitzer, U. Endriss, J. Lang and A.D. Procaccia (eds.). *Handbook of Computational Social Choice* (New York: Cambridge University Press): 399-426.

Geiser, U. (2019). 'Historic Verdict Could Force Swiss Re-Vote on Family Tax Breaks', *Swissinfo.ch*, 10th April, URL = https://www.swissinfo.ch/eng/wrong-statistics_historic-verdict-forces-swiss-re-vote-on-family-tax-breaks/44887174 (accessed 23rd May, 2019).

Grandi, U. and Endriss, U. (2011). 'Binary Aggregation with Integrity Constraints', *Proceedings of the Twenty-Second International Joint Conference on Artificial Intelligence (IJCAI)*: 204-209.

Grandi, U. (2014). 'The Common Structure of Paradoxes in Aggregation Theory', *Computing Research Repository (CoRR)*, URL = https://arxiv.org/abs/1406.2855 (accessed on 20th May, 2019).

Hobolt, S.B. (2006). 'Direct Democracy and European Integration', *Journal of European Public Policy* 13 (1): 153-166.

Kleck, D. (2019). 'Abstimmung Annulliert: Jetzt Giebt es Zwei Möglichkeiten – Eine Wäre für die CVP Klar Besser' ['Referendum Overturned: Now There Are Two Options – One Would Clearly Be Better for the CVP'], *Aargauer Zeitung*, 11th April, URL = https://www.aargauerzeitung.ch/schweiz/abstimmung-annulliert-jetzt-gibt-es-zwei-moeglichkeiten-eine-waere-fuer-die-cvp-klar-besser-134328568 (accessed 23rd May, 2019).

List, C. and Pettit, P. (2002). 'Aggregating Sets of Judgements: An Impossibility Result', *Economics and Philosophy* 18 (1): 89-110.

List, C. and Puppe, C. (2009) 'Judgement Aggregation: a Survey', in P. Anand, P. Pattanaik and C. Puppe (eds.). *The Handbook of Rational and Social Choice* (Oxford: Oxford University Press): 457-482.

North, A. (2019). 'Swiss Courts make facts matter again', *Coda* Story, 23rd April, URL = https://codastory.com/disinformation/swiss-referendum-fake-news/ (accessed 23rd May, 2019).

Nurmi, H. (1997). 'Referendum Design: An Exercise in Applied Social Choice Theory', *Scandinavian Political Studies* 20 (1): 33-52.

Nurmi, H. (1998). 'Voting paradoxes and referenda', *Social Choice and Welfare* 15(3): 333-350.

Nurmi, H. (1999). *Voting Paradoxes and How to Deal with Them* (Berlin and Heidelberg: Springer-Verlag).

Nurmi, H. and Saari, D.G. (2010). 'Connections and Implications of the Ostrogorski Paradox for Spatial Voting Models', in A. van Deemen (ed.), *Collective Decision Making: Views from Social Choice and Game Theory* (Berlin and Heidelberg: Springer-Verlag): 31-56.

Pettit, P. (2012). *On the People's Terms: A Republican Theory and Model of Democracy*. (Cambridge and New York: Cambridge University Press).

Pigozzi, G. (2006). 'Two Aggregation Paradoxes in Social Decision Making: the Ostrogorski Paradox and the Discursive Dilemma' *Episteme* 2 (2): 119-128.

Pitkin, H. F. (1967). *The Concept of Representation* (Berkeley: University of California Press).

Rae, D. and Daudt, H. (1976). 'The Ostrogorski Paradox: A Peculiarity of Compound Majority Decision', *European Journal of Political research* 4 (4): 391-398.

Riker, W. H. (1982). *Liberalism against Populism: A Confrontation between the Theory of Democracy and the Theory of Social Choice.* (San Francisco: W.H. Freeman).

Ostrogorksi, M. (1902). *Democracy and the Organization of Political Parties.* (Volume I-II, New York: Macmillan), translated by F. Clarke. Original: Ostrogorski, M. (1902). *La démocratie et l'organisation des partis politiques.* (Paris: Calmann-Levy).

Voermans, W. (2011). 'Referendum: Geloofsartikel of Hebbeding, Drama of Nuttige Correctie?' ['Referendums: Article of Faith or Must-Have, Drama or Useful Correction?'], *Ars Aequi* 60 (6): 468-478.

Wagner, C. (1983). 'Anscombe's Paradox and the Rules of Three-Fourths' *Theory and Decision* 15 (3): 303-308.

Saari, D. (2008). *Disposing Dictators, Demystifying Voting Paradoxes* (New York: Cambridge University Press).

Schutgens, R.J.B. (2017). 'Referenda', *Rechtsgeleerd Magazijn Themis* 4: 141.

Simpson, E.H. (1951). 'The Interpretation of Interaction in Contingency Tables', *Journal of the Royal Statistical Society*, 1951-Series B (Methodological) : p. 238-241.

Dutch parliamentary papers and Court decisions

Court of The Hague, 12th April 2017, ECLI:NL:RBDHA:2017:3667, rechtspraak.nl, *Stichting Forum voor Democratie* v. *De Staat der Nederlanden*.

Parliamentary Papers (Kamerstukken) II, the Netherlands, 1999/2000, 27034, 4.

Parliamentary Papers (Kamerstukken) II, the Netherlands, 2005/06, 30372, 3.

Supreme Court of the Netherlands, 19th November 1999, ECLI:NL:HR:1999:AA1056, *NJ* 2000/160 (*Tegelen judgement*).