Precise Vibration-Based Damage Localization in 3D Structures Consisting of 1D Elements: Single vs Multiple Response Measurements

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ABSTRACT

The goal of this study is twofold: (i) the refinement of the advanced Functional Model Based Method (FMBM), which through a detailed representation of 3D structures consisting of 1D elements achieves damage precise localization based on single or multiple vibration responses and, (ii) the comparison of the method's effectiveness based on single and multiple response measurements. The refined method is equipped with modified ARX type Functional Models - Vector-dependent Functionally Pooled Vector AutoRegressive with eXogenous excitation (VFP-VARX) models - for the simultaneous exploitation of multiple responses and a proper optimization framework based on which the precise estimation of the damage coordinates upon the 3D structural topology under study is accomplished. The method's effectiveness and the comparison between the use of single or multiple response signals are experimentally assessed via numerous damage cases in a 3D truss structure.

KEYWORDS : *Vibration based SHM, statistical time series methods, damage precise localization, multiple signals, functional models.*

1. INTRODUCTION

Multivariate models that use the information from more than one vibration signals for the modelling of the structural dynamics have been widely used in Structural Health Monitoring (SHM) methods [1–4]. More specifically, these methods may be classified in two main families: (i) those which are based on detailed physical models such as Finite Elements Models [1] and, (ii) those that utilize data-based models such as Neural Networks [2], state space models [3], and Vector AutoRegressive with eXogenous excitation (VARX) type models [4] which are identified exclusively through measured data from the structure.

Data-based methods do not require large and complicated analytical models and treat damage localization as a classification problem where the damage is roughly localized as belonging to a specific number of preselected regions (usually structural elements).This treatment is much simpler than damage precise localization where the estimation of the precise damage coordinates on the investigated structural topology is sought.

The problem of damage precise localization has been investigated within a statistical time series framework through a Functional Model Based Method (FMBM) using scalar (single response) Functional Models [5,6] and very recently through Vector-dependent Functionally Pooled ARX (VFP-ARX) models for precise localization in terms of damage coordinates determination in 3D space [7,8]. The latter type of models offers a global and compact representation of a 3D continuous structural topology under damage at various locations, and may lead to precise localization. In certain cases, however, the localization error based on a single vibration response may be significant [9].

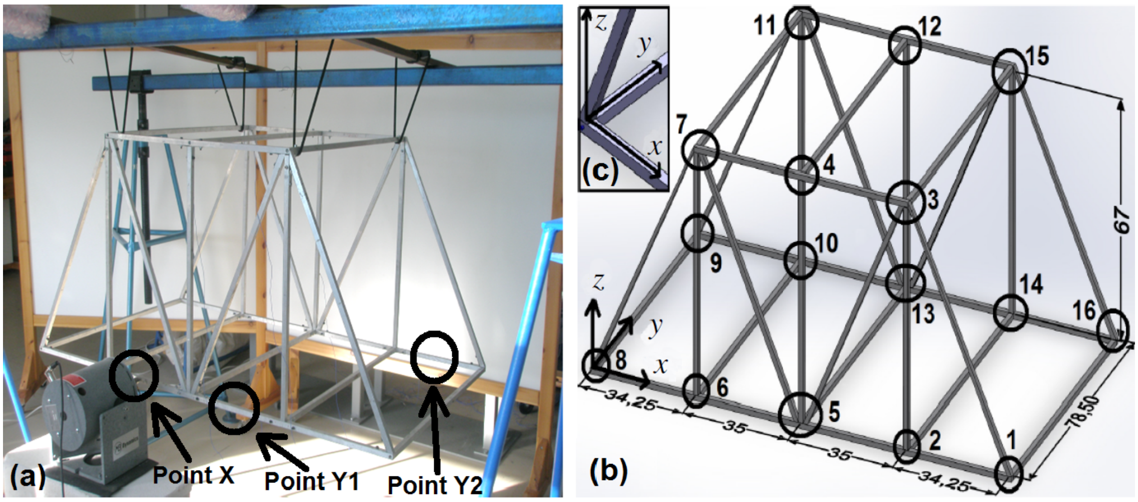

Figure 1 : (a) The truss structure and the experimental setup: The force excitation (Point X) and the two vibration acceleration measurement positions (Point Y1 & Y2), (b) Schematic diagram of the truss. The 16 nodes are indicated by circles (dimensions are in cm). (c) The coordinate system origin is set at node 8.

Thus the goal of the present study is twofold: (i) the refinement of the advanced Functional Model Based Method (FMBM), which through an appropriate optimization framework incorporates the detailed mathematical representation of the structural topology of 3D structures consisting of 1D elements and achieves improved damage localization based on single or multiple vibration responses and, (ii) the comparison of the method's effectiveness based on single and multiple response measurements. The method's refinement necessitates the development of a suitable optimization framework within which the topology of 3D structures consisting of 1D elements is described through analytical geometrical functions as well as the use of multivariate Functional Models, that is VFP-Vector (V) ARX models. These models constitute extensions of the previously used, simpler, VFP-ARX models, and are capable of representing the dynamics, as seen from multiple measurement positions, while also accounting for their interrelations.

The improvement in localization accuracy that is achieved through the refined method is assessed via laboratory experiments with a 3D aluminum truss structure that consists of 16 nodes joined together with 74 bolts. The damage scenarios employed correspond to the loosening of a single bolt at a time, and each damage shall be detected and precisely localized in terms of its Cartesian coordinates. The structure is excited by a random force and the vibration response is measured at two points on the structure via accelerometers. The effectiveness of the presently introduced method and the use of multiple vibration signals is further demonstrated through comparisons with the case where only one response is employed and the previously available method [7].

2. The experimental set-up

The truss structure that is also used in [7] is suspended from metallic beams with a set of elastic cords and hooks (Figure 1(a)) and consists of 26 rods with rectangular cross sections (1.5 $\times$ 1.5 cm) jointed together via steel elbow plates and 74 bolts. All parts are constructed from standard aluminum with the overall dimensions being 140 $\times$ 80 $\times$ 70 cm. Each considered damage corresponds to the complete loosening of a single bolt at a node. The bolts are distributed at 16 nodes as shown in Figure 1(b) and each node has certain coordinates defined based on the origin point ($x = 0, y = 0, z = 0$) taken at node 8 (see Figures 1(b),(c)). Thus, a damage is designated as $F_{x,y,z}$, with x, y, z the coordinates along the corresponding Cartesian axis. Furthermore each rod of the truss is considered as a line without thickness. Damage detection and localization are based on a single-excitation and two

Table 1 : Experimental details.

Structural State	Description	No of experiments
Healthy	-	10 (2 in the baseline phase)
Damaged state	loosening of a bolt at a single location	390 (20 in the baseline phase)
Sampling frequency: $f_s = 128$ Hz		
Signal Bandwidth: $[3-59]$ Hz (bandpass Chebyshev Type II; 12^{th} order)		
Signal length in samples (s): $N = 9758$ (76.23 s)		

vibration acceleration response signals measured via two lightweight accelerometers along y direction at points Y1 & Y2 (Figure 1(a)). The excitation is a random Gaussian force applied horizontally along the y direction at Point X (Figure 1(a)) via an electromechanical shaker equipped with a stinger, and measured via an impedance head. Full details about the equipment and the experimental procedure are found in [7]. A number of experiments are carried out, initially for the healthy structure and subsequently for each damaged state (see details in Table 1).

3. DAMAGE DIAGNOSIS METHODOLOGY

In the current study two methods are used: (a) a Likelihood function based method to its multivariate form for *damage detection* and (b) a refined version of the Functional Model Based Method employing VFP-VARX models and a new optimization framework for improved *damage localization*. Both methods operate within the baseline and inspection phases as presented in the sequel.

Baseline phase. Initially a typical VARX model is estimated based on a single experiment of nx-excitation ny-response signals corresponding to the healthy structure. The model parameter vector is the estimated characteristic quantity based on which damage detection is accomplished via statistical decision making in the framework of the Likelihood function based method [4].

In the following, a VFP-VARX model capable of representing the investigated 3D structural topology is estimated based on a number of experiments from different damage locations on the structure. A total number of $M = M_1 \times M_2 \times M_3$ experiments are performed for a sample of potential damage locations on the considered 3D continuous structural topology, with each experiment being characterized by a specific damage location with coordinates x_l, y_m, z_n. The complete series of experiments cover $x_l \in [x_{min}, x_{max}]$, $y_m \in [y_{min}, y_{max}]$ and $z_n \in [z_{min}, z_{max}]$ via the discretizations $\{x_1, x_2, \ldots, x_{M_1}\}, \{y_1, y_2, \ldots, y_{M_2}\}, \{z_1, z_2, \ldots, z_{M_3}\}$. Thus for a single damage located at a specific point on the 3D structural topology the following *operating parameter vector* $\boldsymbol{k}$ is defined (bold-face upper/lower case symbols designate matrix/column-vector quantities, respectively):

$$\boldsymbol{k} = [x_l \; y_m \; z_n]^T, \;\; l = 1, \ldots, M_1, \;\; m = 1, \ldots, M_2, \;\; n = 1, \ldots, M_3 \tag{1}$$

The complete set of baseline experiments yields a pool of M nx-excitation ny-response signals, each of length N:

$$\boldsymbol{x_k}[t] = \big[x_{1,\boldsymbol{k}}[t] \ldots x_{nx,\boldsymbol{k}}[t]\big]^T_{[nx\times 1]}, \boldsymbol{y_k}[t] = \big[y_{1,\boldsymbol{k}}[t] \ldots y_{ny,\boldsymbol{k}}[t]\big]^T_{[ny\times 1]}$$
$$\text{with } t = 1, \ldots, N, x \in \{x_1, \ldots, x_{M_1}\}, \; y \in \{y_1, \ldots, y_{M_2}\}, z \in \{z_1, \ldots, z_{M_3}\} \tag{2}$$

A mathematical representation of the structural dynamics under *any potential damage location* in the three axis of the Cartesian system, is obtained in the form of a VFP-VARX (na,nb) model of the form:

$$\boldsymbol{y_k}[t] + \sum_{i=1}^{na} \boldsymbol{A}_i(\boldsymbol{k}) \cdot \boldsymbol{y_k}[t-i] = \sum_{i=0}^{nb} \boldsymbol{B}_i(\boldsymbol{k}) \cdot \boldsymbol{x_k}[t-i] + \boldsymbol{e_k}[t], \quad \boldsymbol{e_k}[t] \sim \text{iid}\,\mathcal{N}\big(0, \boldsymbol{\Sigma_e}(\boldsymbol{k})\big) \quad \boldsymbol{k} \in \mathbb{R}^3 \tag{3a}$$

$$\boldsymbol{A}_i(\boldsymbol{k}) = \sum_{j=1}^{p} \boldsymbol{A}_{i,j} \cdot G_j(\boldsymbol{k}), \quad \boldsymbol{B}_i(\boldsymbol{k}) = \sum_{j=1}^{p} \boldsymbol{B}_{i,j} \cdot G_j(\boldsymbol{k}) \tag{3b}$$

with na, nb designating the AutoRegressive (AR) and eXogenous (X) orders, respectively and $\boldsymbol{e_k}[t] = [e_{1,\boldsymbol{k}}[t]\dots e_{ny,\boldsymbol{k}}[t]]^T_{[ny\times 1]}$ the disturbance (innovations) vector that is white (serially uncorrelated) zero-mean with covariance matrix $\boldsymbol{\Sigma_e}(\boldsymbol{k})$ (of dimension $ny \times ny$), potentially cross-correlated with its counterparts corresponding to different experiments.

As Equation (3b) indicates, the AR and X matrix polynomials $\boldsymbol{A}_i(\boldsymbol{k}), \boldsymbol{B}_i(\boldsymbol{k})$ are fully parameterized with dimensions $ny \times ny$ and $ny \times nx$, respectively. They are modelled as explicit functions of the vector $\boldsymbol{k}$ belonging to a p-dimensional functional subspace spanned by the (mutually independent) functions $G_1(\boldsymbol{k}), G_2(\boldsymbol{k}),\dots,G_p(\boldsymbol{k})$ (*functional basis*). The functional basis consist of polynomials of three variables (vector polynomials) obtained as tensor products from univariate polynomials (also see [7]). The matrices $\boldsymbol{A}_{i,j}$, $\boldsymbol{B}_{i,j}$ include the AR and X, coefficients of projection, respectively that are appropriately restructured in vector $\boldsymbol{\theta} = \text{vec}([\boldsymbol{A}_{1,1}\dots\boldsymbol{A}_{na,p} \vdots \boldsymbol{B}_{0,1},\dots\boldsymbol{B}_{nb,p}]^T)$ with dimension $[(na\,ny^2\,p + (nb+1)nx\,ny\,p)\times 1]$ and estimated based on a typical linear regression scheme (see scalar case in [7]) and Ordinary Least Squares (OLS) [10, p. 206].

Inspection phase. A single experiment is conducted under the current, unknown, state of the structure and nx-excitation $\boldsymbol{x}_u[t]$ and ny-response $\boldsymbol{y}_u[t]$ signals are obtained and used for the estimation of a conventional VARX model of the same order with its baseline phase counterpart. *Damage detection* is accomplished by comparing the current model parameter vector with the corresponding from the baseline phase via a statistical hypothesis test which is based on the likelihood function [4].

If the presence of a damage is detected, then *damage localization* is activated based on the FMBM. For this procedure the current signals are driven through the VFP-VARX$(na,nb)_p$ model of the baseline phase which is now re-parametrized with corresponding residuals series $\boldsymbol{e}_u[t,\boldsymbol{k}]$ and covariance matrix $\boldsymbol{\Sigma}_{\boldsymbol{e}_u}$:

$$\mathscr{M}\left(\boldsymbol{k},\boldsymbol{\Sigma}_{\boldsymbol{e}_u}\right) : \boldsymbol{y}_u[t] + \sum_{i=1}^{na} \boldsymbol{A}_i(\boldsymbol{k})\cdot\boldsymbol{y}_u[t-i] = \sum_{i=0}^{nb} \boldsymbol{B}_i(\boldsymbol{k})\cdot\boldsymbol{x}_u[t-i] + \boldsymbol{e}_u[t,\boldsymbol{k}] \tag{4}$$

Based on this re-parametrization the components of $\boldsymbol{k}$ (coordinates of the unknown damage) that lead to the minimum Residual Sum of Squares (RSS) have to be determined through an appropriate optimization procedure that searches on the considered structural topology:

$$\widehat{\boldsymbol{k}}_u = \arg\min_{\boldsymbol{k}} RSS(\boldsymbol{k}) = \arg\min_{\boldsymbol{k}} \sum_{t=1}^{N} \boldsymbol{e}_u^T[t,\boldsymbol{k}]\boldsymbol{e}_u[t,\boldsymbol{k}], \quad \widehat{\boldsymbol{\Sigma}}_{\boldsymbol{e}_u} = \frac{1}{N}\sum_{t=1}^{N} \boldsymbol{e}_u[t,\widehat{\boldsymbol{k}}_u]\boldsymbol{e}_u^T[t,\widehat{\boldsymbol{k}}_u] \tag{5}$$

with $\widehat{\boldsymbol{k}}_u = [x_u\,y_u\,z_u]^T$ and $e_u[t,\boldsymbol{k}]$ given by Equation (4).

A structural element without thickness, that is an 1D element of any shape, in the three dimensional Cartesian system (linear elements are used in the present study) may be expressed by a set of functions of one independent variable s:

$$x = x(s), \quad y = y(s), \quad z = z(s) \tag{6}$$

with $s \in [s_{lb}, s_{ub}]$ defined by the structural topology. Thus $\boldsymbol{k}$ vector may be formulated as a function of the scalar variable s, $\boldsymbol{k}(s) = [x(s)\,y(s)\,z(s)]^T$, and Equation (5) is re-written as:

$$\widehat{s}_u = \arg\min_{s} \sum_{t=1}^{N} \boldsymbol{e}_u^T[t,\boldsymbol{k}(s)]\cdot\boldsymbol{e}_u[t,\boldsymbol{k}(s)], \quad \widehat{\boldsymbol{\Sigma}}_{\boldsymbol{e}_u} = \frac{1}{N}\sum_{t=1}^{N} \boldsymbol{e}_u[t,\boldsymbol{k}(\widehat{s}_u)]\cdot\boldsymbol{e}_u^T[t,\boldsymbol{k}(\widehat{s}_u)] \tag{7}$$

and s_u may be estimated via nonlinear least squares (NLS) estimators [10, pp. 327-329].

The estimate $\widehat{s}_u$ may be shown to be asymptotically Gaussian distributed, with mean equal to the true μ_{s_u} and variance $\sigma^2_{s_u}$ $\left(\widehat{s}_u \sim \mathscr{N}(\mu_{s_u},\sigma^2_{s_u})\right)$ coinciding with the Cramer-Rao lower bound obtained as (for the sake of simplicity $\widehat{\boldsymbol{k}}_u = \boldsymbol{k}(\widehat{s}_u)$):

$$\widehat{\sigma}^2_{s_u} = \frac{1}{N}\left[\frac{1}{4N}\sum_{t=1}^{N}\xi\cdot\boldsymbol{\Lambda}\cdot\widehat{\boldsymbol{\Sigma}}_{\boldsymbol{e}_u}\cdot\boldsymbol{\Lambda}^T\cdot\xi^T\right]^{-1}, \xi = \left.\frac{\partial \boldsymbol{e}_u^T[t,\boldsymbol{k}_u]}{\partial s_u}\right|_{s_u=\widehat{s}_u} = \widehat{\boldsymbol{\theta}}^{\,T}\cdot\left[\boldsymbol{I}_{ny}\otimes\boldsymbol{\varphi}[t]\otimes\left.\frac{\partial \boldsymbol{g}(\boldsymbol{k}_u)}{\partial s_u}\right|_{s_u=\widehat{s}_u}\right] \tag{8}$$

Table 2 : Damage detection and localization details.

Damage Detection - Likelihood function based method

Signal length N	Estimated model	Dimension of $\boldsymbol{\theta}$	No of outputs ny	No of experiments
9758	VARX(62,62)	374	2	2
Estimation method: Ordinary Least Squares (OLS)				
Samples Per Parameter (SPP) = 78.27, Condition number = 3.27 $\times$ 10^{10}				

Damage Localization - FMBM

Signal length N	Estimated model	Dimension of $\boldsymbol{\theta}$	No of outputs ny	No of experiments M
2500	VFP-VARX$(62,62)_{16}$	5984	2	16
Estimation method: Ordinary Least Squares (OLS)				
Samples Per Parameter (SPP) = 20.05, Condition number = 3.18 $\times$ 10^{11}				

with $\boldsymbol{\Lambda} = \widehat{\boldsymbol{\Sigma}}_{\boldsymbol{e}_u}^{-1} + (\widehat{\boldsymbol{\Sigma}}_{\boldsymbol{e}_u}^{T})^{-1}$, $\boldsymbol{I}_{ny}$ designating the $ny \times ny$ unity matrix, $\boldsymbol{\varphi}_{\boldsymbol{k}}[t] = [-\boldsymbol{y}_{\boldsymbol{k}}[t-1] \ldots -\boldsymbol{y}_{\boldsymbol{k}}[t-na] \vdots \boldsymbol{x}_{\boldsymbol{k}}[t] \ldots \boldsymbol{x}_{\boldsymbol{k}}[t-nb]]^T_{[(na\,ny+(nb+1)nx)\times 1]}$ the regressor vector, $\boldsymbol{g}(\boldsymbol{k}_u) = [G_1(\boldsymbol{k}_u) \ldots G_p(\boldsymbol{k}_u)]^T_{[p\times 1]}$ a vector that contains the functional basis and $\otimes$ the Kronecker product [10, pp. 123].

Each damage coordinate v of vector $\widehat{\boldsymbol{k}}_u$ is a random variable – obtained from the appropriate transformation of the Gaussian random variable s according to Equation (6) – that follows a distribution with probability density function (pdf) corresponding to one-to-one transformation given by [11, pp. 197-198]:

$$f_v(v) = f_s(w^{-1}(v)) \cdot \left| \frac{d\,w^{-1}(v)}{d\,v} \right| \tag{9}$$

where $w(\cdot)$ the corresponding function $x(\cdot), y(\cdot), z(\cdot)$ of Equation (6), $w(\cdot)^{-1}$ its inverse and $f_s(s)$ the pdf of s. The mean and variance of v, based on which the confidence intervals of the damage coordinates are constructed, arise through typical expressions [11, pp. 72-73].

In the present study where each rod/element of the truss is considered as a straight line in 3D space without thickness lying between two nodes with coordinates $\{x_0\, y_0\, z_0\}$ and $\{x_1\, y_1\, z_1\}$, respectively, the coordinates of $\widehat{\boldsymbol{k}}_u$ of the corresponding Equation (6) may be expressed as:

$$v = v_0 + \widehat{s}_u (v_1 - v_0) \tag{10}$$

with $v_0 = \{x_0, y_0, z_0\}$ & $v_1 = \{x_1, y_1, z_1\}$ for x_u, y_u, z_u respectively.

Random variable v follows a Gaussian distribution, as it arises from the linear, one-to-one, transformation of the Gaussian variable s, with mean and variance:

$$\mu_v = v_0 + \mu_{s_u}(v_1 - v_0), \quad \sigma_v^2 = (v_1 - v_0)^2 \sigma_{s_u}^2 \tag{11}$$

and corresponding estimates are obtained by replacing $\mu_{s_u} = \widehat{s}_u$ and $\sigma_{s_u}^2 = \widehat{\sigma}_{s_u}^2$.

The obtained estimates of the damage coordinates are accepted as *valid* once the re–parametrized model is successfully validated through typical statistical tests examining the hypothesis of excitation $\boldsymbol{x}_u[t]$ and residual $\boldsymbol{e}_u[t, \widehat{\boldsymbol{k}}_u]$ uncrosscorrelatedness, as well as residual uncorrelatedness (the Mahdi-McLeod whiteness test [12] is used in this study). If the validation procedure is not successful means that the method has converged to a local minimum and needs restarting or that the detected damage is of different type than that the VFP-VARX model is trained to monitor on the continuous structural topology.

4. Damage detection and localization results

Baseline phase. The Likelihood Function based method employs a conventional VARX(62,62) model (characterized by zero excitation delay, $b_0 \neq 0$; Matlab function: *arx.m*) which is estimated based on

Table 3 : Damage detection results.

Method	False Alarms	Missed Damages
Likelihood Function based	0/8	0/390

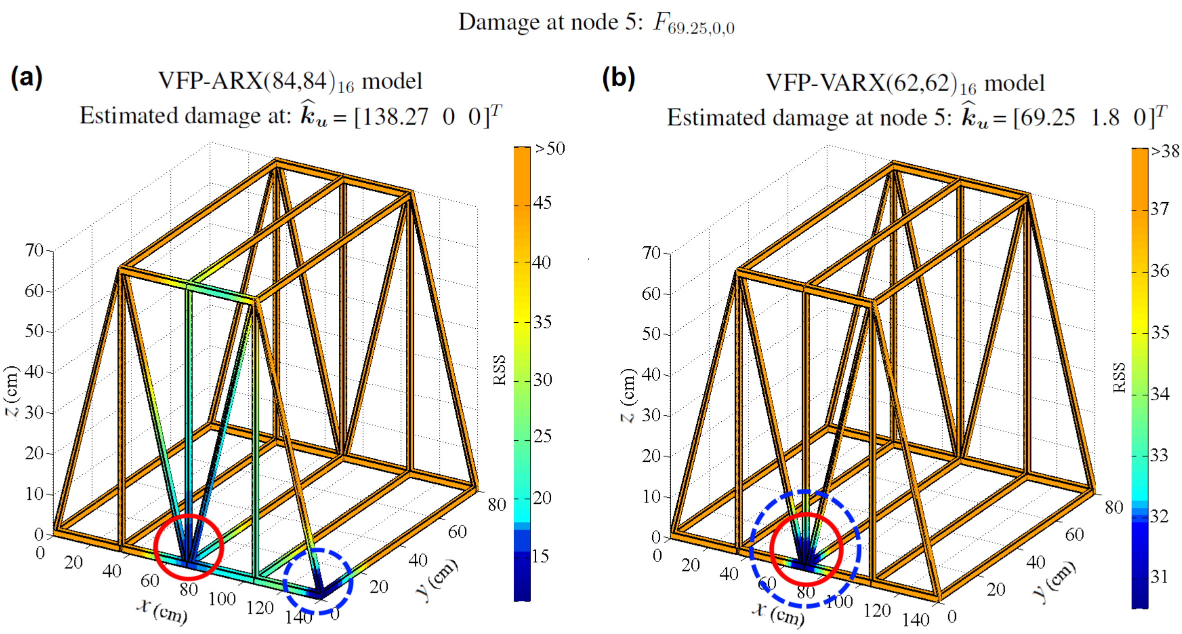

Figure 2 : Indicative damage localization for a single damage case based on: (a) a single acceleration response and the available FMBM; (b) two vibration acceleration responses and the refined FMBM. In each case the actual damage location is shown via a red circle (—), while the estimated location via a blue circle (– –). The precise damage coordinates (actual and estimated) are also indicated above the plots.

a single experiment of $nx = 1$ excitation - $ny = 2$ response signals of the healthy structure. A further experiment with the healthy structure is used for the determination of the statistical threshold. All model identification details are shown in Table 2.

A VFP-VARX(62,62)$_{16}$ model ($b_0 \neq 0$) with functional subspace consisted of $p = 16$ trivariate Shifted Legendre polynomials is estimated based on $M = 16$ experiments of $nx = 1$ excitation - $ny = 2$ response signals for damage localization through the FMBM (see details in Table 2). Model validation is accomplished based on the Mahdi-McLeod residual whiteness test [12] for which a critical point is determined with the use of 20 experiments.

Inspection phase. The $nx = 1$ excitation - $ny = 2$ response signals of a current experiment (unknown structural state) are driven through the conventional VARX(62,62) model of the baseline phase and the obtained residuals are used in the framework of the Likelihood-based function method for damage detection. The damage detection results are presented in Table 3 from which it is evident that the method may identify the healthy condition of the structure without false alarms and missed damages.

Once a damage is detected its precise localization is achieved through the FMBM. The VFP-VARX(62,62)$_{16}$ is thus re-parametrized based on the current signals and the optimization framework (realized via golden search and parabolic interpolation; Matlab function: *fminbnd.m*; tolerance of objective function = 10^{-10}, tolerance of estimated value = 10^{-10}) leads to the corresponding estimates of the unknown damage locations. Figure 2 depicts through an indicative damage case the operation of the optimization framework which searches for the damage location on the truss structure through the minimization of the RSS with respect to variable s. The color code shows the level of RSS with the darkest value to be the minimum point. Evidently, the use of a VFP-VARX(62,62)$_{16}$ model and

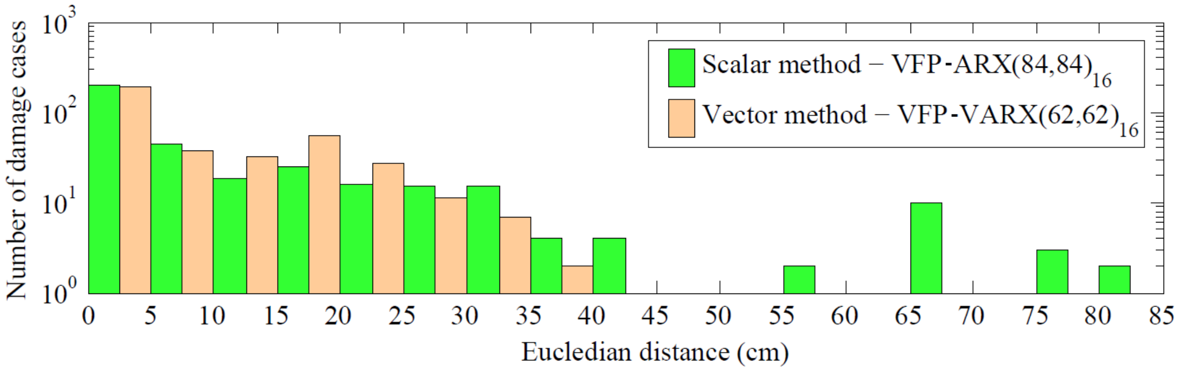

Figure 3 : Damage localization results in terms of the Euclidian distance between the true damage location and its estimate based on single and multiple vibration response measurements (359 damage cases).

multiple response signals compared with the single excitation - single response VFP-ARX$(84,84)_{16}$ of [7] improve significantly the precision of the damage localization. This is also confirmed through the comparison that is presented in Figure 3 in terms of the Euclidian distance between the true and the estimated damage locations based on the scalar and the vector version of method. It is noted that the results of 11 damage cases based on the scalar method with Euclidean distance in the range of 90-130cm are not shown in this figure for the better representation of the comparison between the methods. These cases were greatly improved based on the vector method and moved in the range 20-30cm.

All estimates are successfully validated and corresponding confidence intervals for the damage locations are constructed. Point estimates of the damage coordinates along with the corresponding confidence intervals for 4 indicative damage cases based on the scalar and vector FMBM are presented in Figure 4. It is again evident that the refined method combined with multiple response measurements achieves improved precision in damage localization.

5. Concluding remarks

A refinement of an advanced Functional Model Based Method for damage precise localization that incorporates the detailed mathematical representation of the structural topology of 3D structures consisting of 1D elements in an appropriate optimization framework as well as its assessment based on single and multiple response measurements were presented. The method combined with the exploitation of simultaneous multiple measurements on a truss structure found to achieve precise damage coordinate estimation as well as to be superior over its previous version and the use of a single vibration response. However the method's precision in the estimation of the true damage coordinates may be further improved via an appropriate optimum sensor placement procedure.

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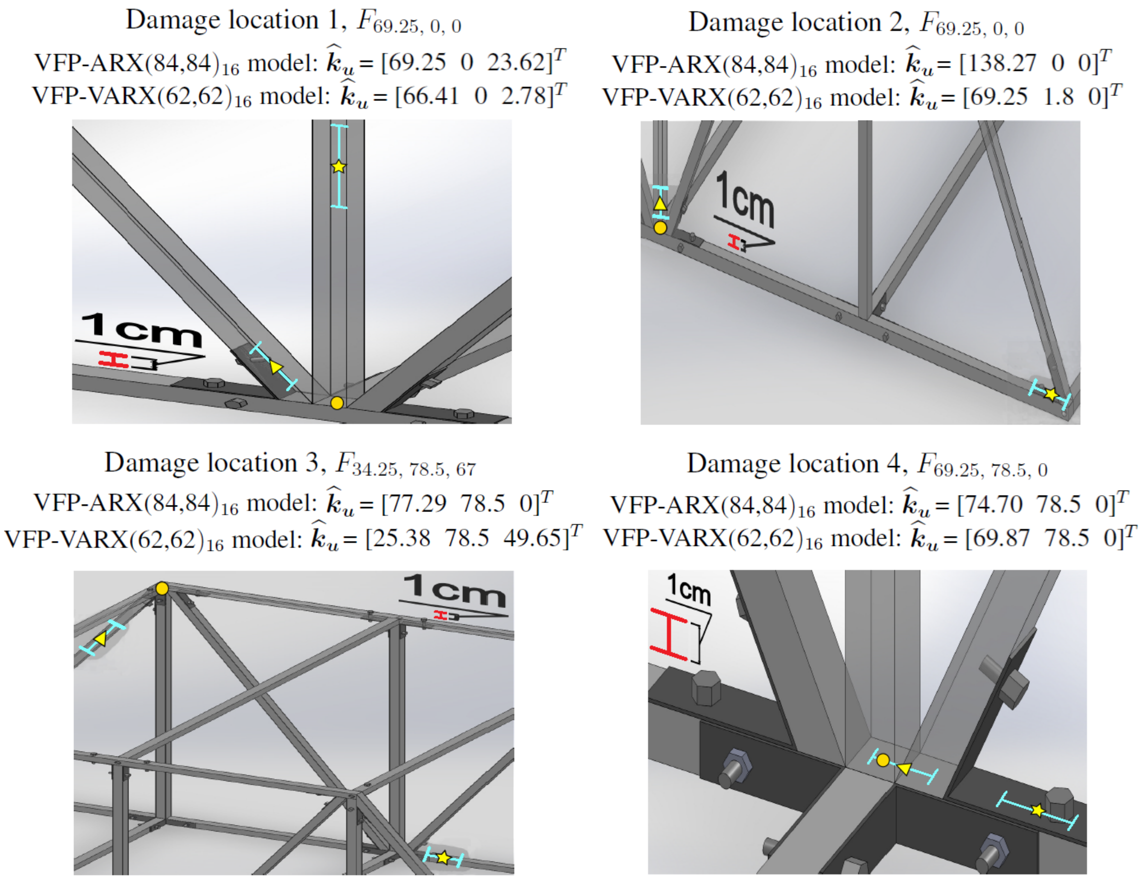

Figure 4 : Indicative damage localization results and comparisons for 4 damage cases (●: true damage location; ▲: point estimate based on the VFP-VARX(62,62)$_{16}$; ★: point estimate based on the VFP-ARX(84,84)$_{16}$; |—|: confidence intervals at the 0.0001 risk level; the true and estimated damage location coordinates are numerically provided above each plot).

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